1. a. Let us first find the present value of the award. This is
\[
\int_{0}^{30} 20000 \times e^{-0.06x} \, dx
\]
Evaluation the integral, we get
\[
-\frac{1}{0.06} \times 20000 \times e^{0.06x}|_{0}^{30} = 278,233
\]
So the prize is worth $278,233. Now, how much money five years from now is worth that? We have to find x such that
\[
x \times e^{-0.06 \times 5} = 278,233
\]
x must be 278,233 \times e^{0.3} = 375,575

b. First we should note that 6\% interest compounded monthly means that after n years, P dollars will equal \( P \times (1 + \frac{0.06}{12})^{12n} \). Alternatively \( P \) dollars \( n \) years from now is worth \( P \times (1 + \frac{0.06}{12})^{-12n} \) dollars today. Since the prize payments start on the winning date and end twenty-nine years from the winning date, the value of the prize is
\[
20000 + 20000(1 + \frac{0.06}{12})^{-12} + 20000(1 + \frac{0.06}{12})^{-12 \times 2} + \ldots + 20000(1 + \frac{0.06}{12})^{12 \times 29}
\]
Setting \( \alpha = (1 + \frac{0.06}{12})^{-12} = .9419 \), we may rewrite this as
\[
20000 \times (1 + \alpha + \alpha^2 + \ldots + \alpha^{29})
\]
Using the formula for the geometric series, we note
\[
1 + \alpha + \alpha^2 + \ldots + \alpha^{29} = \frac{\alpha^{30} - 1}{\alpha - 1} = 14.355
\]
Therefore, the present value of the prize is \( 20,000 \times 14.355 = 287100 \). This corresponded to a payment of \( 287100 \times (1 + \frac{0.06}{12})^{12 \times 5} = 387255 \).
2. a. Use your favorite method (guess and check, separability),

\[ y = Ce^{kx} \]

where \( C \) is some constant.

b. Let us define by \( r(x) \) the radius of a horizontal cross-section of the column \( x \) meters from the top. Therefore the area of the cross-section is \( \pi(r(x))^2 \). This area is proportional to the total mass above it. There is the contribution of the load \( L \) and also the weight of the column. Therefore, the total mass above the cross-section is

\[
L + \int_0^x a \times \pi(r(t))^2 \, dt
\]

where \( a \) is the density of the material. Then the statement about the cross-section and the weight translates to

\[
\pi(r(x))^2 = c(L + \int_0^x a \pi(r(t))^2 \, dt)
\]

Let us find \( c \), the constant of proportionality. Set \( x = 0 \) and we get

\[
\pi r_0^2 = c(L + \int_0^0 a \pi(r(t))^2 \, dt) = cL
\]

Therefore, \( c = \frac{\pi r_0^2}{L} \). For ease of computation, we’ll continue to write \( c \) until the end of the problem. Differentiate the proportionality condition and use the Fundamental Theorem of Calculus to get

\[
2 \pi r(x) r'(x) = ca \pi r(x)^2
\]

Cancelling common terms, we get

\[
r'(x) = \frac{ca}{2} r(x)
\]

This is a differential equation we can solve. Noting \( r(0) = r_0 \) and using part a, we realize the solution must be

\[
r(x) = r_0 e^{\frac{ca}{2} x}
\]

Substituting in \( c \), we get

\[
r(x) = r_0 e^{\frac{\pi a r_0^2}{2L} x}
\]