QUIZ 4
You have 25 minutes.
No notes, no books.
YOU MUST SHOW ALL WORK TO RECEIVE CREDIT
Good luck!

Name _____________________________________________

1. __________ (/20 points)

2. __________ (/20 points)

Total __________ (/40 points)
1. a. After a lawsuit involving people injured by falling ball bearings, Math42ville now has a lottery. It awards a constant stream of cash at $20,000 per year for the following thirty years or $x$ dollars to be made in one payment five years from the winning date. $x$ is chosen so that the two prizes are equivalent. Assuming that interest is 6% compounded continuously, find $x$.

b. Assume that Math42ville elects to dispense with continuously compounded interest and instead uses interest compounded monthly. Assume that the lottery now awards $20,000 in a payment each year for the following thirty years with the first prize payment sent at the winning date or $x$ dollars in one payment five years from the winning date. Find $x$.

Hint: You may use $1 + x + x^2 + x^3 + \ldots + x^n = \frac{x^{n+1} - 1}{x - 1}$
2. a. Solve the differential equation
\[ y' - ky = 0 \]

b. A load \( L \) is supported by a tapered circular column (i.e., a surface of revolution about an axis) whose material has a density \( a \). Suppose the areas of the horizontal cross-sections are proportional to the mass above them. Let \( r_0 \) be the radius of the horizontal cross-section at the very top of the column and \( r(x) \) the radius of a horizontal cross-section a distance \( x \) below the top (that is \( r(0) = r_0 \)). Show that
\[ r(x) = r_0 e^{\pi r_0^2 a x / 2L} \]

Hint: You can find the constant of proportionality by considering the top of the column.