QUIZ 2 SOLUTIONS

1a. Factoring the denominator gives:

\[ x^3 - 2x^2 + x = x(x-1)^2 \]

and so we see that

\[
\frac{1}{x^3 - 2x^2 + x} \, dx = \int A \frac{x}{x} \, dx + \int B \frac{x}{x-1} \, dx + \int \frac{C}{(x-1)^2} \, dx.
\]

1b. We want to use the partial fraction decomposition method. First factor the denominator:

\[ x^2 - 9 = (x+3)(x-3) \]

and so we know that

\[
\frac{12}{x^2 - 9} = \frac{A}{x+3} + \frac{B}{x-3}.
\]

Multiply both sides of the equation by \(x^2 - 9\):

\[ 12 = A(x-3) + B(x+3). \]

By matching coefficients, we see that we need to solve the equations

\[ 0 = A + B \]

and

\[ 12 = -3A + 3B. \]

This is equivalent to \(0 = A + B\) and \(4 = -A + B\). Adding these two equations gives \(4 = 2B\) and so \(B = 2\) and \(A = -2\). Therefore,

\[
\int_0^2 \frac{12}{x^2 - 9} \, dx = \int_0^2 \frac{2}{x+3} \, dx + \int_0^2 \frac{2}{x-3} \, dx.
\]

Integrating the integrals on the right hand side of the equation gives

\[ -2 \ln |x + 3|_0^2 + 2 \ln |x - 3|_0^2 = -2 \ln 5 + 2 \ln 3 + 2 \ln 1 - 2 \ln 3 = -2 \ln 5. \]

2a. Remember that antiderivatives are integrals! We can calculate \( \int x \ln x \, dx \) using integration by parts. Let \( u = \ln x \) and \( dv = x \). Then \( du = dx/x \) and \( v = x^2/2 \), so we have

\[
\int x \ln x \, dx = \frac{x^2 \ln x}{2} - \int \frac{x}{2} \, dx.
\]

Therefore,

\[ F(x) = \frac{x^2 \ln x}{2} - \frac{x^2}{4} + C. \]

Since \( F(1) = 2 \), we see that \( 2 = -1/4 + C \), and so \( C = 2.25 \). Thus, \( F(x) = \frac{x^2 \ln x}{2} - \frac{x^2}{4} + 2.25 \).

2b. Use the substitution \( u = \ln x \):

\[
\int_n^{n^2} \frac{1}{x \ln x} \, dx = \int_{\ln n}^{\ln n^2} \frac{1}{u} \, du = \ln (\ln n^2) - \ln (\ln n).
\]

Now use what you know about logarithms to simplify:

\[ \ln (\ln n^2) - \ln (\ln n) = \ln \left( \frac{\ln (n^2)}{\ln n} \right) = \ln \left( \frac{2 \ln n}{\ln n} \right) = \ln 2. \]