MATH 42
THIRD SAMPLE MIDTERM #1

90 Minutes

NAME:

Section Number:

I agree to abide by the Honor Code.
Signature:

Instructions: Show all work. Unless a numerical approximation is specifically requested, an EXACT solution is required.

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1. Find the following indefinite integrals.

(i) $\int t\sqrt{t^2 - 1} \, dt$

(ii) $\int x \cos(x) \, dx$
(iii) \[ \int \frac{x + 1}{x^2 + 2x - 6} \, dx \]
2. Suppose that we attach one end of a spring to a wall and the other end to a block resting on the (frictionless) floor. If the block is pulled 5 cm from its rest position and then released, Hooke’s law states that the velocity of the block after $t$ seconds is $v(t) = -5\sqrt{k}\sin(\sqrt{k}t)$ cm per second. Here $k$ is a constant that depends on the stiffness of the spring. Suppose that $k = \pi^2/16$.

(a) What is the position of the block after 6 seconds?

(b) How far does the block travel between $t = 1$ and $t = 6$? [Note that we are not looking for the displacement – that is, the change in position. Rather, we want the total distance travelled.]
3. Find the volume of the solid obtained by rotating the region bounded by $y^2 = x - 1$ and $x = 2$ about the $y$-axis.
4. A water storage tank in the shape of the lower half of a sphere of radius 5 feet is buried 7 feet under ground. (That is, the top of the tank is 7 feet under ground.) If the tank is totally full, calculate how much work is required to pump all of the water out. (Note that water weighs 62.4 pounds per cubic foot.)
5. (a) Compute \( \frac{d}{dx} \int_{x^2}^{x^3} e^{\cos t} \, dt \).

(b) Let \( f(x) = \sqrt{1 + x^4} \) and define \( g(x) = \int_a^x f(w) \, dw \). Suppose that, in addition, \( g(3) = 0 \). Prove that \( a = 3 \).
6. Using \( \int_0^\infty e^{-x^2} \, dx = \frac{\sqrt{\pi}}{2} \), compute the following integrals.

(a) \( \int_0^\infty \frac{e^{-x}}{\sqrt{x}} \, dx \)

(b) \( \int_0^\infty x^2 e^{-x^2} \, dx \)
7. Prove that the volume of a pyramid whose base is a square of side length $s$ and whose height is $h$, is $\frac{1}{3}hs^2$. 
8. Find the following limit by expressing it as a definite integral and then using the fundamental theorem of calculus.

\[
\lim_{n \to \infty} \sum_{i=1}^{n} \frac{e^{3(1+\frac{i}{n})}}{n}.
\]