MATH 42
SAMPLE MIDTERM #2

90 Minutes

NAME:

Section Number:

I agree to abide by the Honor Code.
Signature:

Instructions: Show all work. Unless a numerical approximation is specifically requested, an EXACT solution is required.

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1. The use of ozone depleting chemicals is reducing the ozone in the upper atmosphere by 0.25% annually. How long will it take for 10% of the ozone to be destroyed?
2. (a) Find the general solution to $y' = 2x\sqrt{y - 1}$.

(b) Solve the initial value problem $y' = 4x^3y - y$, $y(1) = 3$. 
(c) Find the general solution to \( \frac{d^2y}{dx^2} = 2 \frac{dy}{dx} - 2y. \)

(d) Solve the initial value problem \( \frac{d^2y}{dx^2} = \frac{dy}{dx} + 2y \) with \( y(0) = 0, \) \( y'(0) = 2. \)
3. Two chemicals, $U$ and $V$, combine to form a third, $X$; one gram of $U$ combines with 2 grams of $V$ to form 3 grams of $X$. Suppose that 50 grams of $U$ is allowed to combine with 100 grams of $V$. The chemicals combine at a rate proportional to the product of the untransformed masses. Write a differential equation to model how much substance $X$ is present after $t$ seconds. Also, assuming that after 1 second 75 grams of chemical $X$ has been produced, write a formula for the amount of $X$ produced after $t$ seconds.
4. A cylindrical buoy of diameter 1 meter and mass 100 kg floats vertically in water. If it is pushed down and released, it oscillates. Find the period of this oscillation. You may assume that water is 1000 \( \text{kg/m}^3 \) and that the acceleration due to gravity is \( g = 10 \text{m/sec}^2 \). (Note that a body in water is bouyed up by a force equal to the weight of the water it displaces.)
5. The math 42 disease spreads at a rate proportional the product of the number of people who have the disease and the number that do not yet have it. At a certain university, 14 thousand of the 20 thousand students currently have the math 42 disease, and that number is growing at 420 students per week. Suppose that we begin a regimen of therapy that cures 480 students per week but confers no immunity. Determine what will happen in the long run and explain how you know.
6. Two companies, Banana and Kumquat, are competing in the same market. Let $B(t)$ and $K(t)$ be the annual revenues (in millions of dollars) of Banana and Kumquat, respectively. The following equations describe the rates of growth of these companies:

$$\frac{dB}{dt} = 3B - 2B^2 - BK$$

$$\frac{dK}{dt} = 2K - 2BK - K^2$$

(a) In the long run, what would happen to Banana’s revenue if Kumquat did not exist? What would happen to Kumquat’s revenue if Banana did not exist?

(b) What happens in the long run if both Banana and Kumquat have positive revenue?
7. (a) A 200 gallon tank is filled with 90% water and 10% alcohol. A mixture of 99% water and 1% alcohol is poured into the tank at the rate of 1 gallon per minute, and liquid from the tank is drained off at the same rate. How long does it take for the tank to become 95% water?

(b) Suppose that as liquid is drained out of the tank in part (a), it is pumped into a second tank with a capacity of 300 gallons. Liquid leaves the second tank at the same rate so it remains full at all times. Write a differential equation satisfied by the amount of alcohol in the second tank.

(c) What can you say about the amount of alcohol in the second tank after a long time? Briefly explain how you know.