QUIZ 2 SOLUTIONS

1a. In order for a function \( f \) to be continuous at a point \( a \), the limit \( \lim_{x \to a} f(x) \) must be equal to \( f(a) \). In the above function, we see that

\[
f(2) = 2^{2k-3}.
\]

We need to choose \( k \) appropriately so that \( \lim_{x \to a} f(x) \) exists. We compute the left and right hand limits separately. The left hand limit is

\[
\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} \frac{x - \sqrt{x + 2}}{3x - 6} = \lim_{x \to 2^-} \frac{(x - \sqrt{x + 2})(x + \sqrt{x + 2})}{(3x - 6)(x + \sqrt{x + 2})} = \lim_{x \to 2^-} \frac{x^2 - (x + 2)}{3x - 6(x + \sqrt{x + 2})} = \lim_{x \to 2^-} \frac{x - 2}{3(x - 2)(x + \sqrt{x + 2})} = \lim_{x \to 2^-} \frac{x + 1}{3(x + \sqrt{x + 2})} = \frac{3}{12} = \frac{1}{4}
\]

On the other hand, the right hand limit is just

\[
\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} 2^{kx-3} = 2^{2k-3} = f(2),
\]

and so we need to solve

\[
2^{2k-3} = \frac{1}{4}.
\]

Since \( \frac{1}{4} = 2^{-2} \), we see that \( 2k - 3 = -2 \) or \( k = \frac{1}{2} \).

1b. Because \( f \) is even, our graph must be symmetric about the y-axis. We must have a horizontal asymptote at \( y = 2 \), and vertical asymptotes at \( x = 2, x = -2, x = 4 \) and \( x = -4 \).

2a. The statement \( \lim_{x \to a} f(x) = L \) means that for every \( \epsilon > 0 \), you can find a \( \delta > 0 \) so that \( |f(x) - L| < \epsilon \) whenever \( 0 < |x - a| < \delta \).

2b. We need to find \( \delta \) so that \( |x^2 - 6x + 9| < \epsilon \) whenever \( 0 < |x - 3| < \delta \). Since \( x^2 - 6x + 9 = (x - 3)^2 \), we see that if we take \( \delta = \sqrt{\epsilon} \), then that choice of \( \delta \) “works;” that is, if \( |x - 3| < \sqrt{\epsilon} \), then \( |x - 3|^2 = |x^2 - 6x + 9| < (\sqrt{\epsilon})^2 = \epsilon \).