

# EXAM 2

Math 216, 2021 Spring.

Name: \_\_\_\_\_ NetID: \_\_\_\_\_ Student ID: \_\_\_\_\_

## GENERAL RULES

YOU MUST SHOW ALL WORK AND EXPLAIN ALL REASONING TO RECEIVE CREDIT.  
CLARITY WILL BE CONSIDERED IN GRADING.

No calculators.

All answers must be reasonably simplified.

All of the policies and guidelines on the class webpages are in effect on this exam.

It is strongly advised that you use black pen only, since that will be most clear in scanning your work.

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## DUKE COMMUNITY STANDARD STATEMENT

“I have adhered to the Duke Community Standard in completing this examination.”

Signature: \_\_\_\_\_

1. (16 pts) Find a fundamental set of real solutions to the constant coefficient linear differential equation  $L(y) = 0$  whose characteristic polynomial is

$$p(\lambda) = (\lambda + 1)(\lambda - 2)^3(\lambda^3 + 5\lambda^2 + 17\lambda + 13)^2$$

2. (17 pts) Find the form of a particular solution to the constant coefficient linear differential equation

$$L(y) = 6xe^x \sin(3x) - x^2e^{4x}$$

whose characteristic polynomial is

$$p(\lambda) = (\lambda - 4)^7(\lambda^2 + 5)(\lambda^2 - 2\lambda + 10)(4\lambda + 2)$$

3. (16 pts) In this question we have a 1 kg mass, and a hanging spring which stretches 2 m when the mass is attached. Gravitational acceleration in this question is  $10 \text{ m/s}^2$  and there is no friction.

(a) Find a fundamental set of real solutions to the differential equation describing the above system.

(b) Suppose now that an external periodic force of  $\cos(4t)$  N is applied. Find a particular solution and identify if there is resonance. (Recall  $1 \text{ N} = 1 \text{ kg m/s}^2$ .)

4. (17 pts) Let  $V$  be the vector space of continuously differentiable vector fields in  $\mathbb{R}^3$ , let  $B$  be the solid closed unit ball in  $\mathbb{R}^3$ , let  $g$  be a continuous real-valued function on  $\mathbb{R}^3$ , and define  $P_g : V \rightarrow \mathbb{R}$  by

$$P_g(\vec{F}) = \iiint_B (\nabla \cdot \vec{F}(\vec{x})) (g(\vec{x})) dV$$

Bob says that  $P_g$  is a linear transformation; prove he is right, or find a counterexample to show he is wrong.

5. (17 pts) There are values of  $b$  and  $c$  such that for every  $y$  that is a solution to the differential equation

$$y^{[5]} + 6y^{[4]} + 11y''' + 10y'' + 13y' + 3y = 0$$

then  $by' + cy$  is a solution to the differential equation

$$y^{[4]} + 3y''' + 2y'' + 4y' + y = 0.$$

Use linear differential operators to find these values, WITHOUT plugging in directly OR solving any system of equations.

6. (17 pts) In  $P_2$  we consider the bases  $\mathcal{S} = \{1, x, x^2\}$  and  $\mathcal{V} = \{1, (x-2), (x-2)^2\}$ . Let  $T : P_2 \rightarrow P_2$  be the linear transformation defined by  $T(p)(x) = \frac{d}{dx}((x-2)p(x))$  (so, given a polynomial  $p$ , multiply by  $(x-2)$  and then take the derivative to get  $T(p)$ ).

(a) Find the matrix  $M = [T]_{\mathcal{V}}^{\mathcal{V}}$ .

(b) Find the change of basis matrices  $[I]_{\mathcal{V}}^{\mathcal{S}}$  and  $[I]_{\mathcal{S}}^{\mathcal{V}}$ .

(c) Use the results from the previous two parts to find the matrix  $A = [T]_{\mathcal{S}}$ .