EXAM 1

Math 216, 2021 Spring.

Name:__________________________________ NetID:___________ Student ID:___________

GENERAL RULES

YOU MUST SHOW ALL WORK AND EXPLAIN ALL REASONING TO RECEIVE CREDIT. CLARITY WILL BE CONSIDERED IN GRADING.

No calculators.

All answers must be reasonably simplified.

All of the policies and guidelines on the class webpages are in effect on this exam.

It is strongly advised that you use black pen only, since that will be most clear in scanning your work.

DUKE COMMUNITY STANDARD STATEMENT

"I have adhered to the Duke Community Standard in completing this examination."

Signature: _______________________

1. *(18 pts)* The matrix A is

$$\begin{pmatrix} 1 & 2 & 0 \\ 0 & 2 & 4 \\ 2 & 6 & 4 \end{pmatrix}$$

(a) Find a matrix E for which EA is the reduced row echelon form R for A.

(b) Find the complete set of solutions to the system $A\vec{x} = (3, 2, 8)$.

(c) We are interested in finding a vector $\vec{c}$ for which $A\vec{x} = E^{-1}\vec{c}$ has no solutions. Explain a simple strategy (NOT guess-and-check!) that would lead to finding such a vector.

2. *(15 pts)* The matrix A is the product BC, where B is 3×5 and C is 5×3. B is known to have two identical rows. Show that A is not invertible. (Explain your reasoning with all details!)

3. *(18 pts)* Use a row reduction to find the inverse of A, the determinant of A, and to write A as a product of elementary matrices.

$$A = \begin{pmatrix} 0 & 2 & 1 \\ 0 & 1 & 5 \\ 1 & 0 & 1 \end{pmatrix}$$

4. *(15 pts)* Find a basis for the plane $x + 2y + 3z = 0$ thought of as a subspace of $\mathbb{R}^3$, and show that it is a basis without making any use of dimension.

5. *(18 pts)*

(a) Describe what it would mean for the list of functions below to be linearly dependent, using an equation explicitly involving those functions and a description of what must be true about that equation.

$$\{4\sin(x-\pi/5), 2\cos(x-\pi/7), 3\sin(x-\pi/11)\}$$

(b) The general fact that $[c_1 v_1 + \ldots + c_n v_n]_\beta = c_1[v_1]_\beta + \ldots + c_n[v_n]_\beta$ allows for rewriting the equation above as equivalent to an equation involving m vectors in $\mathbb{R}^k$. What are these values of m and k, and an example of such a basis β?

(c) Give a rank argument to conclude whether the original list is linearly independent or linearly dependent.

6. *(16 pts)*

(a) Find the Wronskian of $\{3\sin^2 x, 4\cos^2 x, 2\}$ by direct computation from the definition. What can you conclude ONLY from the result of this computation? Is there other information about these functions individually that combined with the result of the above computation allows you to conclude more?

(b) Bob claims to have computed the Wronskian of $\{\sin^2 x, \cos^2 x, \tan^2 x, \sec^2 x\}$, and that his result is

$$w(x) = \sin^2 x \cos^4 x + \tan^2 x \sec^4 x$$

Is he right? Explain fully.