EXAM 1

Math 216, 2020 Fall.

Name:____________________________________ NetID:____________ Student ID:____________

GENERAL RULES

YOU MUST SHOW ALL WORK AND EXPLAIN ALL REASONING TO RECEIVE CREDIT. CLARITY WILL BE CONSIDERED IN GRADING.

No calculators.

All answers must be reasonably simplified.

All of the policies and guidelines on the class webpages are in effect on this exam.

It is strongly advised that you use black pen only, since that will be most clear in scanning your work.

DUKE COMMUNITY STANDARD STATEMENT

"I have adhered to the Duke Community Standard in completing this examination."

Signature: ________________________

(Scratch space. Nothing on this page will be graded!)

1. *(20 pts)*

(a) Bob is doing a row operation of a matrix. At some point he has a matrix with rows R_1, R_2, R_3, and he is contemplating as his next "step" to replace the first row with $R_1 + 2R_2$, the second row with $R_2 + 2R_3$, and the third row with $4R_3 - R_1$. What can you tell Bob about the advisability of this?

(b) Suppose that the matrix A has the existence property but does not have the uniqueness property. Explain in detail how you know that A^T cannot have the existence property. *(Hint: What can you conclude about the number of rows and the number of columns?)*

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2. *(20 pts)*

(a) The 3×3 matrix $B = A^{-1}$ has rows B_1, B_2, B_3, and columns $\vec{b}_1$, $\vec{b}_2$, $\vec{b}_3$. Find (by describing either its rows or columns) the inverse of $C = PA$, where

$$P = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

(b) Find elementary matrices $E_1, \ldots, E_k$ such that

$$E_1 \cdots E_k = \begin{pmatrix} 1 & 3 \\ 2 & 5 \end{pmatrix}$$

(extra space for questions from other side)

3. *(16 pts)* The curve C in the xy-plane has equation $(2x - 3y)^2 + (x + y)^2 = 1$. Find (using ideas from this course!) the area enclosed by C. *(Hint: Find a matrix that relates this to the unit circle in the uv-plane.)*

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4. *(24 pts)* We will say that a vector space X "splits" the spaces V and W if either $V \subsetneq X \subsetneq W$ or $W \subsetneq X \subsetneq V$. ("$P \subsetneq Q$" means that P is a subspace of Q and is not equal to Q.)

   (a) Is there a vector space C that splits $A = \mathbb{R}^3$ and $B = \{\text{the } x\text{-axis in } \mathbb{R}^3\}$? If so, find it (you don't have to prove your claim); if not, explain why it can't exist.

   (b) Is there a vector space F that splits $D = \text{span}\{\sin x - \cos x,\ 3\sin x + 2\cos x,\ 2\sin x - \cos x\}$ and $E = \text{span}\{\sin x,\ \cos x,\ e^x,\ e^{-x}\}$? If so, find it (you don't have to prove your claim); if not, explain why it can't exist.

   (c) Suppose that $V \subsetneq W$ are finite-dimensional. On what condition does there exist a vector space that splits them? (You don't have to prove your claim.)

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5. *(20 pts)* In this question we consider vectors in the vector space

$$V = \left\{ \begin{pmatrix} f_1 \\ f_2 \end{pmatrix} \middle| f_1, f_2 \text{ are real-valued functions on } \mathbb{R} \right\}$$

(a) Show that $\alpha = \left\{ \begin{pmatrix} \sin x \\ 0 \end{pmatrix}, \begin{pmatrix} \cos x \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ \sin x \end{pmatrix}, \begin{pmatrix} 0 \\ \cos x \end{pmatrix} \right\}$ is a basis for its span W.

(b) Decide if $\beta = \left\{ \begin{pmatrix} \sin x - \cos x \\ \sin x + \cos x \end{pmatrix}, \begin{pmatrix} \cos x \\ \sin x + \cos x \end{pmatrix}, \begin{pmatrix} \sin x + \cos x \\ \sin x \end{pmatrix} \right\}$ is linearly independent.

(Show how you use that $[av + bw]_{\mathcal{V}} = a[v]_{\mathcal{V}} + b[w]_{\mathcal{V}}$, and $([v]_{\mathcal{V}} = \vec{0}) \Leftrightarrow (v = 0)$.)

(extra space for questions from other side)