EXAM 3

Math 216, 2020 Spring, Clark Bray.

Name:____ Section:______ Student ID:____________

GENERAL RULES

YOU MUST SHOW ALL WORK AND EXPLAIN ALL REASONING TO RECEIVE CREDIT. CLARITY WILL BE CONSIDERED IN GRADING.

No calculators.

All answers must be reasonably simplified.

All of the policies and guidelines on the class webpages are in effect on this exam.

WRITING RULES

Use black pen only. You may use a pencil for initial sketches of diagrams, but the final sketch must be drawn over in black pen and you must wipe all erasure residue from the paper.

DUKE COMMUNITY STANDARD STATEMENT

"I have adhered to the Duke Community Standard in completing this examination."

Signature: ________________________

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1. *(18 pts)* The basis $\mathcal{V}$ consists of $\vec{v}_1 = (6, 2, 3)$, $\vec{v}_2 = (-3, 6, 2)$, and $\vec{v}_3 = (2, 3, -6)$. The linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ is the reflection through the plane P spanned by the vectors $\vec{v}_1$ and $\vec{v}_2$.

   (a) Find the change of basis matrix $[I]^{\mathcal{S}}_{\mathcal{V}}$.

   (b) Find the change of basis matrix $[I]^{\mathcal{V}}_{\mathcal{S}}$. *(Hint: What feature of $\mathcal{V}$ will help with this?)*

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(c) Use the results from the previous two parts to find the matrix $[T]_{\mathcal{S}}^{\mathcal{S}}$.

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2. *(16 pts)* The non-diagonalizable matrix A has eigenvalues 2 and 3 with multiplicities 1 and 2, respectively. It can be put into Jordan form by way of the basis $\mathcal{V}$ consisting of $\vec{v}_1 = (1, 0, 1)$, $\vec{v}_2 = (0, 1, 2)$, and $\vec{v}_3 = (0, 0, 2)$ (in this order).

   (a) Given only the information above, what are the possible Jordan forms for A?

   (b) Suppose we know that $\vec{v}_2$ is an eigenvector. Find the only possible Jordan form for A.

   (c) Find the matrix A itself.

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3. *(16 pts)* The vectors $v_1, \ldots, v_n$ in the inner product space V are known to be orthonormal. Show that these vectors are linearly independent. *(Hint: Write a relation, and take clever inner products.)*

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4. *(18 pts)* Find the complete set of solutions to

$$\vec{y}' = \begin{pmatrix} 17 & -12 \\ 20 & -14 \end{pmatrix} \vec{y} + \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

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5. *(16 pts)* Use the basis consisting of the vectors $\vec{v}_1 = (1,1)$ and $\vec{v}_2 = (0,1)$ to derive a back-solvable system $\vec{z}' = B\vec{z}$ (do not solve!) whose solutions would allow for solving the system

$$\begin{aligned} y_1' &= y_1 + y_2 \\ y_2' &= -y_1 + 3y_2 \end{aligned}$$

and derive a formula for solutions $\vec{y}(t)$ in terms of those solutions $\vec{z}(t)$.

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6. *(16 pts)*

(a) If the equation below were converted to a first order system $\vec{y}' = A\vec{y}$, what would be the characteristic polynomial of the coefficient matrix A?

$$y''' - 3y'' + 2y = 0$$

(b) Find a first order system of linear differential equations whose solutions would allow also for identifying the solutions to the system below.

$$\begin{aligned} y_1''' &= y_1 - 2y_2 + 3y_1' + 4y_2'' \\ y_2''' &= 5y_1 + 6y_1' + 7y_1'' - 8y_2 \end{aligned}$$

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