## EXAM 3

Math 216, 2020 Spring, Clark Bray.

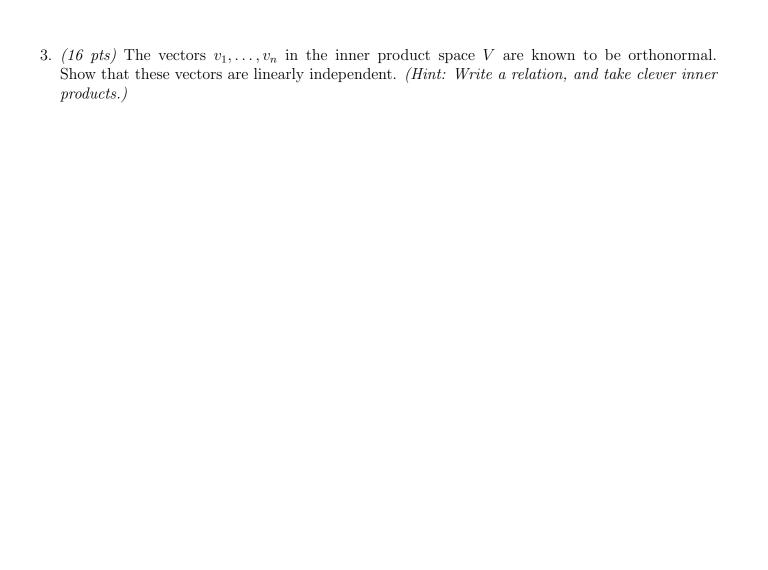
Name:	_ Section:	Student ID:
GENERAL R	RULES	
YOU MUST SHOW ALL WORK AND EXPLAIN AL CLARITY WILL BE CONSIDERED IN GRADING.	LL REASONING	G TO RECEIVE CREDIT.
No calculators.		
All answers must be reasonably simplified.		
All of the policies and guidelines on the class webpages	s are in effect on	this exam.
WRITING R	RULES	
Use black pen only. You may use a pencil for initial skedrawn over in black pen and you must wipe all erasure		
DUKE COMMUNITY STAN	NDARD STAT	EMENT
"I have adhered to the Duke Community Stan	ndard in complet	ing this examination."
Signature:		

- 1. (18 pts) The basis  $\mathcal{V}$  consists of  $\vec{v}_1 = (6,2,3)$ ,  $\vec{v}_2 = (-3,6,2)$ , and  $\vec{v}_3 = (2,3,-6)$ . The linear transformation  $T: \mathbb{R}^3 \to \mathbb{R}^3$  is the reflection through the plane P spanned by the vectors  $\vec{v}_1$  and  $\vec{v}_2$ .
  - (a) Find the change of basis matrix  $[I]_{\mathcal{V}}^{\mathcal{S}}$ .

(b) Find the change of basis matrix  $[I]_{\mathcal{S}}^{\mathcal{V}}$ . (Hint: What feature of  $\mathcal{V}$  will help with this?)

(c)	(c) Use the results from the previous two parts to find the n	natrix $[T]_{\mathcal{S}}^{\mathcal{S}}$ .

2.	(16 pts) The non-diagonalizable matrix $A$ has eigenvalues 2 and 3 with multiplicities 1 and 2, respectively. It can be put into Jordan form by way of the basis $\mathcal V$ consisting of $\vec v_1=(1,0,1)$ , $\vec v_2=(0,1,2)$ , and $\vec v_3=(0,0,2)$ (in this order).
	(a) Given only the information above, what are the possible Jordan forms for $A$ ?
	(b) Suppose we know that $\vec{v}_2$ is an eigenvector. Find the only possible Jordan form for A.
	(c) Find the matrix $A$ itself.



4.  $(18 \ pts)$  Find the complete set of solutions to

$$\vec{y}' = \begin{pmatrix} 17 & -12 \\ 20 & -14 \end{pmatrix} \vec{y} + \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

5. (16 pts) Use the basis consisting of the vectors  $\vec{v}_1 = (1,1)$  and  $\vec{v}_2 = (0,1)$  to derive a back-solvable system  $\vec{z}' = B\vec{z}$  (do not solve!) whose solutions would allow for solving the system

$$y_1' = y_1 + y_2$$
  
 $y_2' = -y_1 + 3y_2$ 

and derive a formula for solutions  $\vec{y}(t)$  in terms of those solutions  $\vec{z}(t)$ .

- 6. (16 pts)
  - (a) If the equation below were converted to a first order system  $\vec{y}' = A\vec{y}$ , what would be the characteristic polynomial of the coefficient matrix A?

$$y''' - 3y'' + 2y = 0$$

(b) Find a first order system of linear differential equations whose solutions would allow also for identifying the solutions to the system below.

$$y_1''' = y_1 - 2y_2 + 3y_1' + 4y_2''$$
  
$$y_2''' = 5y_1 + 6y_1' + 7y_1'' - 8y_2$$