EXAM 2

Math 216, 2020 Spring, Clark Bray.

Name:________________________________ Section:______ Student ID:____________

GENERAL RULES

YOU MUST SHOW ALL WORK AND EXPLAIN ALL REASONING TO RECEIVE CREDIT. CLARITY WILL BE CONSIDERED IN GRADING.

No notes, no books, no calculators.

All answers must be reasonably simplified.

All of the policies and guidelines on the class webpages are in effect on this exam.

WRITING RULES

Do not write anything near the staple – this will be cut off.

Use black pen only. You may use a pencil for initial sketches of diagrams, but the final sketch must be drawn over in black pen and you must wipe all erasure residue from the paper.

Work for a given question can be done ONLY on the front or back of the page the question is written on. Room for scratch work is available on the back of this cover page, and on the two blank pages at the end of this packet; scratch work will NOT be graded.

DUKE COMMUNITY STANDARD STATEMENT

"I have adhered to the Duke Community Standard in completing this examination."

Signature: ________________________

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1. *(16 pts)*

(a) What is the value $w(1)$ of the Wronskian of the list of functions below defined on $[-1, 1]$? *(Reminder – be sure to explain all of your reasoning!)*

$$\sin(x), \csc\left(x - \frac{\pi}{2}\right), \sin\left(x - \frac{\pi}{3}\right), \cos\left(x - \frac{\pi}{4}\right)$$

(b) It is known that the Wronskian of the linearly independent list y_1, y_2, y_3, y_4 is zero for all values of x. Does this information allow us to draw any additional conclusions, and if so, what is it that we can conclude?

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2. *(18 pts)* Let V be the collection of all functions of the form $f(x) = M\cos(x - \phi)$, where M and ϕ can be any real values.

   (a) Show that V is a vector space. *(Hint: Rewrite it as a span and cite a theorem about spans.)*

   (b) Identify a basis for V and compute the coordinates with respect to that basis of $g(x) = \cos(x - \pi/3)$.

(extra space for questions from other side)

3. *(16 pts)* Find a fundamental set of real solutions to the differential equation $L(y) = 0$ whose characteristic polynomial is $p(\lambda) = (\lambda - 4)^3(\lambda^2 + 4)^2(\lambda^3 + 6\lambda^2 + 11\lambda + 6)$.

(extra space for questions from other side)

4. *(16 pts)* Find the form of a particular solution to the differential equation

$$y'' - 4y' + 29y = 3x + xe^{2x}\cos(5x) - x^2e^{4x}\sin(5x)$$

(extra space for questions from other side)

5. *(16 pts)* A mass on a spring in a resistive medium is currently critically damped. Using certain units, it is known that the mass is 3 and the spring constant is 27.

   (a) What is the friction coefficient of this mass in this resistive medium?

   (b) What is the (smallest) increase in the mass that would result in a (decaying) oscillation with frequency equal to $1/2\pi$?

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6. *(18 pts)* Let V be the vector space of solutions to the differential equation $y'' - y' + y = 0$.

(a) Show that $T : V \to \mathbb{R}^2$ defined by $T(f) = \begin{pmatrix} f(0) \\ f'(0) \end{pmatrix}$ is a linear transformation.

(b) What theorem allows us to conclude that T is bijective? Check that the conditions of this theorem apply to the given differential equation.

(c) What does the bijectivity of T allow us to conclude about the dimension of V? (Explain!)

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