EXAM 1

Math 216, 2019 Fall, Clark Bray.

Name:______________________________________ Section:______ Student ID:____________

GENERAL RULES

YOU MUST SHOW ALL WORK AND EXPLAIN ALL REASONING TO RECEIVE CREDIT. CLARITY WILL BE CONSIDERED IN GRADING.

No notes, no books, no calculators.

All answers must be reasonably simplified.

All of the policies and guidelines on the class webpages are in effect on this exam.

WRITING RULES

Do not write anything near the staple – this will be cut off.

Use black pen only. You may use a pencil for initial sketches of diagrams, but the final sketch must be drawn over in black pen and you must wipe all erasure residue from the paper.

Work for a given question can be done ONLY on the front or back of the page the question is written on. Room for scratch work is available on the back of this cover page, and on the two blank pages at the end of this packet; scratch work will NOT be graded.

DUKE COMMUNITY STANDARD STATEMENT

"I have adhered to the Duke Community Standard in completing this examination."

Signature: ________________________

(Scratch space. Nothing on this page will be graded!)

1. *(18 pts)* Bob says that a complete row reduction (to its RREF, R) of the nonsingular matrix A is performed by left multiplication by the matrix G below.

$$G = \begin{pmatrix} 1 & 3 & 2 \\ 0 & 1 & 5 \\ 0 & 2 & 9 \end{pmatrix}$$

(a) Is G actually a combination of row operations, or not?

(b) Find the matrix A^{-1} or show that it does not exist.

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2. *(18 pts)* The reduced row echelon form for A is

$$R = \begin{pmatrix} 1 & 3 & 0 & 4 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

(a) Find the complete set of solutions to $A\vec{x} = \vec{0}$.

(b) Suppose A times the vector $(1, 2, 3, 4)$ is $(5, 6, 7)$. Find the complete set of solutions to $A\vec{x} = (5, 6, 7)$.

(extra space for questions from other side)

3. *(14 pts)* Use the arithmetic below to compute the determinant of the matrix M below *without* using cofactors or a row reduction.

$$\det\begin{pmatrix}1 & 3 & 2\\ 2 & 2 & 1\\ 3 & 1 & 1\end{pmatrix} = -4, \quad \det\begin{pmatrix}1 & 1 & 2\\ 2 & 1 & 1\\ 3 & 1 & 1\end{pmatrix} = -1, \quad M = \begin{pmatrix}1 & 6 & 2\\ 2 & 5 & 1\\ 3 & 4 & 1\end{pmatrix}$$

(extra space for questions from other side)

4. *(20 pts)* Bob is hoping to be able to use vector space theorems to study L, the set of all positive-real-valued functions defined on $\mathbb{R}$, with operations $\oplus$ and $\otimes$ defined by

$$\begin{aligned}(f \oplus g)(x) &= f(x)g(x) \\ (c \otimes f)(x) &= |c|f(x)\end{aligned}$$

(a) Is the condition below satisfied? Prove or find a counterexample

$$u + (v + w) = (u + v) + w \text{ for all vectors } u, v, w$$

(b) Is L a vector space? Prove or show a condition that is not satisfied.

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5. *(15 pts)* Show that the plane $3x - 2y + z = 0$ in $\mathbb{R}^3$ is a vector space.

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6. *(15 pts)* Suppose that p_1, p_2, p_3, p_4 are all quadratic (at most) polynomials. Without using the notion of dimension, show that this list must be linearly dependent.

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