EXAM 3

Math 216, 2019 Spring, Clark Bray.

Name:________________________________ Section:______ Student ID:____________

GENERAL RULES

YOU MUST SHOW ALL WORK AND EXPLAIN ALL REASONING TO RECEIVE CREDIT. CLARITY WILL BE CONSIDERED IN GRADING.

No notes, no books, no calculators.

All answers must be reasonably simplified.

All of the policies and guidelines on the class webpages are in effect on this exam.

WRITING RULES

Do not write anything on the QR codes or nearby print, or near the staple.

Use black pen only. You may use a pencil for initial sketches of diagrams, but the final sketch must be drawn over in black pen and you must wipe all erasure residue from the paper.

Work for a given question can be done ONLY on the front or back of the page the question is written on. Room for scratch work is available on the back of this cover page, and on the two blank pages at the end of this packet; scratch work will NOT be graded.

DUKE COMMUNITY STANDARD STATEMENT

"I have adhered to the Duke Community Standard in completing this examination."

Signature: ________________________

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1. *(20 pts)* The function $T : P_2 \to P_3$ is a linear transformation, and $T(f)$ is defined as the unique antiderivative F of f with $F(0) = 0$. Let $\mathcal{S}_2 = \{1, x, x^2\}$, $\mathcal{S}_3 = \{1, x, x^2, x^3\}$, $\mathcal{V} = \{1+x, x+x^2, x^2\}$, $\mathcal{W} = \{1 - x + 3x^2, x + x^2 - x^3, x^2 - 4x^3, 4x^3\}$.

   (a) Compute $M = [T]_{\mathcal{S}_2}^{\mathcal{S}_3}$.

   (b) Compute $[I]_{\mathcal{V}}^{\mathcal{S}_2}$ and $[I]_{\mathcal{W}}^{\mathcal{S}_3}$.

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(c) The matrix $[T]_{\mathcal{V}}^{\mathcal{W}}$ can be written as QMR. Find either Q or Q^{-1} (whichever you prefer), and find either R or R^{-1} (whichever you prefer).

(d) Suppose we know that $[T(f)]_{\mathcal{W}} = \begin{pmatrix} 0 \\ 2 \\ 1 \\ 1 \end{pmatrix}$. Compute $[T]_{\mathcal{S}_2}^{\mathcal{S}_3}[f]_{\mathcal{S}_2}$ *without* finding f explicitly.

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2. *(15 pts)* Find the diagonalization (the resulting diagonal matrix AND the basis used to achieve it) of the matrix A below.

$$A = \begin{pmatrix} 8 & -10 \\ 3 & -3 \end{pmatrix}$$

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3. *(15 pts)* For v, w, x in the inner product space V, we are given the following information:

$$\begin{pmatrix} \langle v, v\rangle & \langle v, w\rangle & \langle v, x\rangle \\ & \langle w, w\rangle & \langle w, x\rangle \\ & & \langle x, x\rangle \end{pmatrix} = \begin{pmatrix} 5 & 2 & 3 \\ & 5 & 3 \\ & & 3 \end{pmatrix}$$

Compute the angle between $v + w$ and x in V.

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4. *(15 pts)* The basis $\mathcal{V}$ for $\mathbb{R}^3$ consists of the three vectors below.

$$\vec{v}_1 = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ 0 \end{pmatrix} \quad \vec{v}_2 = \begin{pmatrix} \frac{1}{\sqrt{6}} \\ \frac{-1}{\sqrt{6}} \\ \frac{2}{\sqrt{6}} \end{pmatrix} \quad \vec{v}_3 = \begin{pmatrix} \frac{1}{\sqrt{3}} \\ \frac{-1}{\sqrt{3}} \\ \frac{-1}{\sqrt{3}} \end{pmatrix}$$

Find the coordinates of $\vec{x} = \begin{pmatrix} 3 \\ 2 \\ 4 \end{pmatrix}$ with respect to $\mathcal{V}$.

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5. *(20 pts)* Find a fundamental set of real solutions to the system $\vec{y}' = M\vec{y}$.

$$M = \begin{pmatrix} 2 & -2 \\ 5 & 0 \end{pmatrix}$$

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6. *(15 pts)* Find a fundamental set of real solutions to the system $\vec{y}' = B\vec{y}$.

$$B = \begin{pmatrix} 6 & 9 \\ -1 & 0 \end{pmatrix}$$

(Hint: $(3, -1)$ *is the only eigenvector. You may find a convenient use for the vector* $(1, 0)$*.)*

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