EXAM 2

Math 216, 2019 Spring, Clark Bray.

Name:____ Section:______ Student ID:____________

GENERAL RULES

YOU MUST SHOW ALL WORK AND EXPLAIN ALL REASONING TO RECEIVE CREDIT. CLARITY WILL BE CONSIDERED IN GRADING.

No notes, no books, no calculators.

All answers must be reasonably simplified.

All of the policies and guidelines on the class webpages are in effect on this exam.

WRITING RULES

Do not write anything on the QR codes or nearby print, or near the staple.

Use black pen only. You may use a pencil for initial sketches of diagrams, but the final sketch must be drawn over in black pen and you must wipe all erasure residue from the paper.

Work for a given question can be done ONLY on the front or back of the page the question is written on. Room for scratch work is available on the back of this cover page, and on the two blank pages at the end of this packet; scratch work will NOT be graded.

DUKE COMMUNITY STANDARD STATEMENT

"I have adhered to the Duke Community Standard in completing this examination."

Signature: __________________________

(Scratch space. Nothing on this page will be graded!)

1. *(20 pts)* The matrix A with columns $\vec{a}_1$, $\vec{a}_2$, $\vec{a}_3$, $\vec{a}_4$ has reduced row echelon form R below.

$$R = \begin{pmatrix} 1 & 0 & 2 & 4 \\ 0 & 1 & 3 & 5 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

(a) Find a relation among the vectors $\vec{a}_1$, $\vec{a}_2$, $\vec{a}_4$.

(b) Find a basis for the row space of A.

(c) Find a basis for the column space of A.

(extra space for question from other side)

2. *(10 pts)* The functions f_1, f_2, f_3 are given below. Decide if this trio is linearly independent.

$$\begin{aligned} f_1(x) &= 3\sin(x) + 1\cos(x) + 2\cos\left(x - \frac{\pi}{3}\right) \\ f_2(x) &= 1\sin(x) + 4\cos(x) + 1\cos\left(x - \frac{\pi}{3}\right) \\ f_3(x) &= 2\sin(x) + 2\cos(x) + 5\cos\left(x - \frac{\pi}{3}\right) \end{aligned}$$

(extra space for question from other side)

3. *(20 pts)* Your friend Bob asks you to help him find all possible solutions to the differential equation below on the interval $(0, \infty)$, and he knows somehow that the three functions x, x^2, and x^3 are each related to the answer.

$$x^2y'' - 4xy' + 6y = 10x$$

(a) Bob's first thought is to try to find roots of the characteristic polynomial. Is this a good way to start? Explain.

(b) Identify a relevant linear differential operator L and apply it to each of the three functions given above.

(c) Use the results of the previous part to find the general solution.

(extra space for question from other side)

4. *(20 pts)* Find a fundamental set of real solutions to the constant coefficient linear differential equation whose characteristic polynomial is below.

$$p(\lambda) = (\lambda - 4)^3(\lambda^2 + 2\lambda + 26)^2$$

(extra space for question from other side)

5. *(10 pts)* Find all of the cube roots (in rectangular form) of $8i$.

(extra space for question from other side)

6. *(20 pts)* Prove that the kernel of a linear transformation $T : V \to W$ must be a subspace of V.

(extra space for question from other side)

(Scratch space. Nothing on this page will be graded!)

(Scratch space. Nothing on this page will be graded!)

(Scratch space. Nothing on this page will be graded!)

(Scratch space. Nothing on this page will be graded!)