EXAM 2

Math 216, 2018 Fall, Clark Bray.

Name:________________________________ Section:______ Student ID:____________

GENERAL RULES

YOU MUST SHOW ALL WORK AND EXPLAIN ALL REASONING TO RECEIVE CREDIT. CLARITY WILL BE CONSIDERED IN GRADING.

No notes, no books, no calculators. Scratch paper is allowed, but (1) it must be from the instructor, (2) it must be returned with the exam, and (3) it will NOT be graded.

All answers must be reasonably simplified.

All of the policies and guidelines on the class webpages are in effect on this exam.

WRITING RULES

Do not write anything on the QR codes or near the staple.

Use black pen only. You may use a pencil for initial sketches of diagrams, but the final sketch must be drawn over in black pen and you must wipe all erasure residue from the paper.

Work for a given question can ONLY be done on the front or back of the page the question is written on.

DUKE COMMUNITY STANDARD STATEMENT

"I have adhered to the Duke Community Standard in completing this examination."

Signature: ________________________

(Nothing on this page will be graded!)

1. *(20 pts)* The reduced row echelon form of the matrix A is given below.

$$\begin{pmatrix} 1 & 2 & 0 & 5 \\ 0 & 0 & 1 & 6 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

(a) Find a basis for the null space of A.

(b) Find a basis for the row space of A.

(c) The matrix A has columns $\vec{a}_1, \vec{a}_2, \vec{a}_3, \vec{a}_4$ (in that order). Find a significant relation among these columns of A and explain how you arrived at your answer.

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2. *(15 pts)* Bob is interested in deciding if the list of functions $\{\cos x + \sin x, \sin x + x\cos x, x\cos x + x\sin x, x\sin x + \cos x\}$ is linearly independent or linearly dependent. He has begun a computation of the Wronskian, and has correctly derived it to:

$$\det \begin{pmatrix} \cos x + \sin x & \sin x + x\cos x & x\cos x + x\sin x & x\sin x + \cos x \\ -\sin x + \cos x & 2\cos x - x\sin x & \cos x - x\sin x + \sin x + x\cos x & x\cos x \\ -\cos x - \sin x & -3\sin x - x\cos x & -2\sin x - x\cos x + 2\cos x - x\sin x & \cos x - x\sin x \\ \sin x - \cos x & -4\cos x + x\sin x & -3\cos x + x\sin x - 3\sin x - x\cos x & -2\sin x - x\cos x \end{pmatrix}$$

Understandably, he very much does not want to work out this determinant. Can you help him to use the Wronskian to decide if this list is linearly independent? (Be sure that your answer makes significant use of the Wronskian. Explain all of your reasoning.)

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3. *(15 pts)* Find a fundamental set of real solutions to the linear differential equation $L(y) = 0$ whose partially factored characteristic polynomial is given below.

$$p(\lambda) = (\lambda + 2)^2(\lambda^3 - 3\lambda + 2)(\lambda^2 + 6\lambda + 25)^2$$

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4. *(15 pts)* Find the form of a particular solution to the equation $L(y) = x^2e^x - xe^{-x}\cos(2x)$, whose partially factored characteristic polynomial is given below.

$$p(\lambda) = (\lambda^2 + 2\lambda + 5)^3$$

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5. *(15 pts)* Find the gain and phase shift in the physical system represented by the differential equation below.

$$y'' + y' + 3y = 2\cos(2t)$$

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6. *(20 pts)* The linear transformation $T : P_4 \to C^0$ has

$$T(1) = \sin x, \quad T(x) = e^x, \quad T(x^2) = 4\sin x - e^x, \quad T(x^3) = e^x - \sin x, \quad T(x^4) = 3e^x + 2\sin x$$

(a) Find $T\Big((x+1)^3\Big)$.

(b) Find $\dim(\mathrm{im}(T))$.

(c) Find $\dim(\ker(T))$.

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