EXAM 2

Math 216, 2017-2018 Spring, Clark Bray.

You have 50 minutes.

No notes, no books, no calculators.

YOU MUST SHOW ALL WORK AND EXPLAIN ALL REASONING
TO RECEIVE CREDIT. CLARITY WILL BE CONSIDERED IN GRADING.
All answers must be simplified. All of the policies and guidelines
on the class webpages are in effect on this exam.

Good luck!

Name ______________________________

Disc.: Number__________ TA____________________ Day/Time____________________

"I have adhered to the Duke Community Standard in completing this examination."

Signature: ____________________

1. __________

2. __________

3. __________

4. __________

5. __________

6. __________

Total Score __________ (/100 points)

1. *(16 pts)* Show that the image of the linear transformation $T : V \to W$ is a subspace of W.

2. *(18 pts)*

(a) Find bases for the null space, row space, and column space of the matrix

$$A = \begin{pmatrix} 1 & 0 & 5 & 3 \\ 0 & 1 & 7 & 4 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

(b) The $x \times y$ matrix B has a 3-dimensional row space and a 5-dimensional null space. What can you conclude about x and/or y, if anything? What can you conclude about the dimension of the column space, if anything?

3. *(18 pts)* Find a fundamental set of real solutions to the differential equation $L(y) = 0$ whose partially factored characteristic polynomial is given below.

$$p(\lambda) = (\lambda^3 + 5\lambda^2 + 9\lambda + 5)^3$$

4. *(16 pts)* Find the form of a particular solution to the differential equation below. (You do not have to evaluate the coefficients.)

$$y'' + 2y' + 10y = xe^{-x}\cos(3x) + x^2e^{4x}\sin(2x)$$

5. *(16 pts)* For the differential equation below representing a mass on a spring with a sinusoidal external force, a particular solution is $y_p = -\cos(3t)$.

$$y'' + 8y = \cos(3t)$$

Using this, derive

(a) the general homogeneous solution;

(b) the IVP solution with $y(0) = 0$ and $y'(0) = 0$,

(c) the beat frequency (f, not ω) of this IVP solution (as an arithmetic expression – not a decimal approximation).

6. *(16 pts)* Let $S = \text{span}(\cos(4x), \sin(4x))$ be the vector space of solutions to the differential equation $y'' + 16y = 0$, and let I be the vector space of initial values $I = \left\{ \begin{pmatrix} y(0) \\ y'(0) \end{pmatrix} \right\} = \left\{ \begin{pmatrix} k_1 \\ k_2 \end{pmatrix} \right\} = \mathbb{R}^2$.

   Show that the function $T : S \to I$ defined by $T(y) = \begin{pmatrix} y(0) \\ y'(0) \end{pmatrix}$ is a linear transformation.