

EXAM 2

Math 216, 2017-2018 Fall, Clark Bray.

You have 50 minutes.

No notes, no books, no calculators.

YOU MUST SHOW ALL WORK AND EXPLAIN ALL REASONING
TO RECEIVE CREDIT. CLARITY WILL BE CONSIDERED IN GRADING.

All answers must be simplified. All of the policies and guidelines
on the class webpages are in effect on this exam.

Good luck!

Name _____

Disc.: Number _____ TA _____ Day/Time _____

“I have adhered to the Duke Community
Standard in completing this
examination.”

1. _____

2. _____

3. _____

4. _____

5. _____

6. _____

Signature: _____

Total Score _____ (/100 points)

1. (16 pts)

(a) Use the Wronskian to decide if the list below is linearly independent or linearly dependent.

$$x^2 - 1, \quad 3x + 2, \quad 2x^2 + 3x$$

(b) Use any technique from this course to decide if the list below is linearly independent or linearly dependent.

$$3x^2e^{x^3} + 2 \sin x \cos^2 x + 2 \ln(3 + \sin x), \quad 1x^2e^{x^3} + 3 \sin x \cos^2 x + 2 \ln(3 + \sin x), \\ 6x^2e^{x^3} + 8 \sin x \cos^2 x + 1 \ln(3 + \sin x), \quad 5x^2e^{x^3} + 1 \sin x \cos^2 x + 5 \ln(3 + \sin x)$$

2. (18 pts) Find a fundamental set of real solutions to the differential equation $L(y) = 0$ whose characteristic polynomial is

$$p(\lambda) = (\lambda + 2)^3(\lambda^2 + 5)(\lambda^2 + 6\lambda + 10)^2(\lambda^3 + 4\lambda^2 + 5\lambda + 2)$$

3. (16 pts) Find the form of a particular solution to the differential equation below. (You do not have to evaluate the unknown constants.)

$$y''' + 2y'' + 4y' + 8y = x \sin(2x) + \cos(2x)$$

4. (16 pts) Find a particular solution to the differential equation below.

$$y' + y = e^{-x} \cos 3x$$

5. (16 pts) Bob says that the function $S : C^\infty \rightarrow \mathbb{R}$ defined below acts linearly. Is he right? If he is, prove it; if he is not, find an explicit counterexample.

$$S(f) = 3f'(0) + 2f(0) + 1$$

6. (18 pts)

(a) Show that $(D - 3)(x^k e^{3x}) = kx^{k-1}e^{3x}$ when $k \geq 1$.

(b) The differential operator $L : C^\infty \rightarrow C^\infty$ is defined by

$$L = (D - 3)^3(D - 1)(D^2 - 4D + 6)$$

Without citing theorems from section 4.2, show directly that $x^2 e^{3x}$ is in $\ker(L)$.

(c) The differential operator $Q : C^\infty \rightarrow C^\infty$ is defined by

$$Q = (D - 1)(D^2 - 4D + 6)$$

(Note the relationship between this Q and the operator L on the previous page.) The function $4x^5e^{3x}$ is a solution to the differential equation $Q(y) = g(x)$.

Without citing theorems from section 4.3, find a solution to the differential equation $L(y) = g(x)$ of the form Ax^8e^{3x} .