EXAM 1

Math 216, 2017-2018 Fall, Clark Bray.

You have 50 minutes.

No notes, no books, no calculators.

YOU MUST SHOW ALL WORK AND EXPLAIN ALL REASONING
TO RECEIVE CREDIT. CLARITY WILL BE CONSIDERED IN GRADING.
All answers must be simplified. All of the policies and guidelines
on the class webpages are in effect on this exam.

Good luck!

Name ______________________________

Disc.: Number__________ TA____________________ Day/Time____________________

"I have adhered to the Duke Community Standard in completing this examination."

Signature: ____________________

1. __________
2. __________
3. __________
4. __________
5. __________
6. __________

Total Score __________ (/100 points)

1. *(21 pts)* We consider the matrix A below.

$$A = \begin{pmatrix} 1 & 1 & 2 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 3 & 3 & 1 \end{pmatrix}$$

(a) Find a matrix M for which $MA = R$ is the reduced row echelon form of A.

(b) Find the complete set of solutions to the system $A\vec{x} = \vec{b}$, with $\vec{b} = (1, 2, 0)$.

(c) Compute the determinant of M *without* using a cofactor expansion.

2. *(16 pts)* Bob is doing a row reduction of a matrix with three rows, and in the interests of saving time is combining row operations – but is worried he might possibly be combining too many at a time. At one point, he contemplates doing a step as indicated below.

$$\left.\vphantom{\begin{array}{c}1\\1\\1\end{array}}\right) \begin{array}{c} 3\textcircled{1} - 2\textcircled{3} \\ 5\textcircled{2} + \textcircled{1} + \textcircled{3} \\ 2\textcircled{2} - 4\textcircled{3} \end{array}$$

Can you help Bob decide if this actually is a combination of row operations?

3. *(16 pts)* We consider the matrix B below.

$$B = \begin{pmatrix} 1 & 3 & 0 \\ 0 & 6 & 0 \\ 2 & 5 & 4 \end{pmatrix}$$

(a) Compute the determinant of B using a cofactor expansion.

(b) Compute the determinant of B *without* using a cofactor expansion, instead using antisymmetry and an established theorem about triangular matrices.

4. *(15 pts)* Let D be the set of all diagonal 3×3 matrices with non-negative entries, with addition $\oplus$ and scalar multiplication $\otimes$ defined by

$$A \oplus B = AB$$

$$c \otimes \begin{pmatrix} a_1 & 0 & 0 \\ 0 & a_2 & 0 \\ 0 & 0 & a_3 \end{pmatrix} = \begin{pmatrix} a_1^c & 0 & 0 \\ 0 & a_2^c & 0 \\ 0 & 0 & a_3^c \end{pmatrix}$$

(a) Show that D does satisfy the following condition: "There is a zero vector, 0, with $0 + v = v$ for all $v \in V$."

(b) Show that D does satisfy the following condition: "$c(u + v) = cu + cv$ for all $u, v \in V$, $c \in \mathbb{R}$."

(c) Explain fully why D is not a vector space.

5. *(16 pts)* Let P be the collection of polynomials with roots at $x = 0$ and $x = 1$ (and possibly elsewhere), with the usual addition and scalar multiplication. Is P a vector space? Prove or disprove.

6. *(16 pts)* The functions f, g, and h are known to be linearly independent. Decide if the list $f - g$, $g - h$, $f + 2g + 3h$ is linearly independent or linearly dependent.