EXAM 2

Math 216, 2016-2017 Spring, Clark Bray.

You have 50 minutes.

No notes, no books, no calculators.

YOU MUST SHOW ALL WORK AND EXPLAIN ALL REASONING
TO RECEIVE CREDIT. CLARITY WILL BE CONSIDERED IN GRADING.
All answers must be simplified. All of the policies and guidelines
on the class webpages are in effect on this exam.

Good luck!

Name ___________________________________

Disc.: Number____________ TA_______________________ Day/Time_______________________

"I have adhered to the Duke Community
Standard in completing this
examination."

Signature: _______________________

1. ____________

2. ____________

3. ____________

4. ____________

5. ____________

6. ____________

Total Score ____________ (/100 points)

1. *(16 pts)* We consider here the collection S of all functions of the form

$$ae^{x+b} + ce^{3x+d}$$

(a) Show that S is a vector space.

(b) Find the dimension of S.

2. *(18 pts)* Consider the following arguments that Bob makes using the Wronskian. In each case:

   I. if his conclusions are invalid, explain why they are invalid, and if possible what might fix them.

   II. if his conclusions are valid, identify what parts of his arguments (if any) are unnecessary.

   (a) For the functions $f_1, \ldots, f_4$, Bob computes correctly that $w(x) = \sin x$. He also notes correctly that these functions are analytic. From these facts he concludes that the list of functions is linearly independent.

   (b) For the functions $g_1, \ldots, g_6$, Bob computes correctly that $w(x) = 0$. He also notes correctly that these functions are analytic. From these facts he concludes that the list of functions is linearly dependent.

   (c) For the functions $p_1, \ldots, p_5$ (all of which are degree 4 polynomials), Bob notes correctly that all degree 4 polynomials can be written as linear combinations of them. He also notes correctly that they are all solutions to a linear differential equation $L(y) = 0$ whose coefficient functions are continuous. He then concludes (i) that he does not yet have enough information to decide if this list of functions is linearly independent, (ii) that he should thus proceed to compute the Wronskian, and (iii) that if he finds a value x_0 with $w(x_0) = 0$ then he could conclude the list was linearly dependent.

3. *(16 pts)* Find a fundamental set of real solutions to the differential equation whose partially factored characteristic polynomial is below.

$$p(\lambda) = (\lambda - 4)^3(\lambda^3 + 4\lambda^2 + \lambda - 6)(\lambda - (2+3i))^2(\lambda - (2-3i))^2$$

4. *(16 pts)* Find a particular solution to the differential equation below.

$$y' + 3y = \sin 2x$$

5. *(16 pts)* In a chosen set of units, the motion of a mechanical system is modeled by the differential equation

$$y'' + fy' + 9y = 4\cos 2t$$

Find an expression for the gain in the system in terms of the friction coefficient f. (Compute this directly as the amplitude of the output divided by the amplitude of the input – do not cite shortcuts from examples from the notes.) What value of f maximizes the gain?

6. *(18 pts)* Consider the four dimensional vector space $V = \text{span}\{\sin x, \cos x, e^{-x}\sin x, e^{-x}\cos x\}$, and the linear transformation $T : V \to C^0$ given by

$$T(f) = f'' + 2f' + 2f$$

(a) Find the images of the four given vectors spanning V.

(b) Use the result of part (a) to compute the dimension of $\text{im}(T)$.

(c) Use the result of part (b) to compute the dimension of $\ker(T)$.