EXAM 1

Math 216, 2016-2017 Spring, Clark Bray.

You have 50 minutes.

No notes, no books, no calculators.

YOU MUST SHOW ALL WORK AND EXPLAIN ALL REASONING
TO RECEIVE CREDIT. CLARITY WILL BE CONSIDERED IN GRADING.
All answers must be simplified. All of the policies and guidelines
on the class webpages are in effect on this exam.

Good luck!

Name ______________________________

Disc.: Number__________ TA____________________ Day/Time____________________

"I have adhered to the Duke Community Standard in completing this examination."

Signature: ____________________

1. __________

2. __________

3. __________

4. __________

5. __________

6. __________

Total Score __________ (/100 points)

1. *(16 pts)* Find the complete set of solutions to the system below.

$$\begin{aligned} x - 3z &= -2 \\ 2y + z &= 5 \\ x + 2y - 2z &= 3 \end{aligned}$$

2. *(16 pts)* Find the unique values of c and k that make the system below have infinitely many solutions (x, y, z). (As always, don't forget to explain your reasoning!)

$$\begin{aligned} 2x + y + cz &= k \\ x + y + 5z &= 5 \\ y + 2z &= 4 \end{aligned}$$

(Hint: Find c first, and then make convenient use of the following nonsingular matrix:)

$$\begin{pmatrix} 0 & 1 & -1 \\ -2 & 4 & -1 \\ 1 & -2 & 1 \end{pmatrix}$$

3. *(18 pts)* The 3×3 matrix A is row reduced to the identity matrix by the following sequence of row operations:

   i. The first row is multiplied by 2.
   ii. 4 times the first row is added to the second row.
   iii. The second row is divided by 3.
   iv. 2 times the second row is added to the third row.
   v. The third row is multiplied by 5.

   (a) Compute the determinant of A.

   (b) Find elementary matrices $E_1, \ldots, E_5$ such that $A = E_1 E_2 E_3 E_4 E_5$.

4. *(16 pts)* Bob says he likes to live dangerously when it comes to "combining row operations". At one point in a row reduction, he takes the following "step":

$$\left)\begin{array}{l} 3\textcircled{1} + 2\textcircled{2} \\ 2\textcircled{2} - 3\textcircled{1} + 4\textcircled{3} \\ 4\textcircled{3} + 2\textcircled{1} \end{array}\right.$$

(a) Is Bob's above "step" equivalent to left multiplication by some matrix? If so, what is the matrix?

(b) Is Bob's "step" actually a combination of row operations? (If so, you are not required to identify the row operations; but in either case you must justify your reasoning.)

5. *(16 pts)* The following determinants are given (and easily checked).

$$\det\begin{pmatrix} 233 & 1 & 234 \\ 234 & 0 & 235 \\ 233 & 0 & 234 \end{pmatrix} = -1 \qquad \det\begin{pmatrix} 233 & 0 & 234 \\ 234 & 1 & 235 \\ 233 & 0 & 234 \end{pmatrix} = 0 \qquad \det\begin{pmatrix} 233 & 0 & 234 \\ 234 & 0 & 235 \\ 233 & 1 & 234 \end{pmatrix} = 1$$

Show how you can use this information to compute the determinant below.

$$\det\begin{pmatrix} 233 & 126 & 234 \\ 234 & 511 & 235 \\ 233 & 376 & 234 \end{pmatrix}$$

6. *(18 pts)* We are interesed in the question of whether the list of vectors below is linearly independent or linearly dependent.

$$\vec{v}_1 = \begin{pmatrix} 2 \\ 1 \\ 3 \\ 6 \end{pmatrix}, \quad \vec{v}_2 = \begin{pmatrix} 1 \\ 3 \\ 1 \\ 2 \end{pmatrix}, \quad \vec{v}_3 = \begin{pmatrix} 0 \\ 2 \\ 5 \\ 6 \end{pmatrix}$$

(a) Show a step-by-step argument, starting from the definition of linear dependence, that results in a matrix equation whose solution(s) would allow for resolving the question.

(b) Suppose you knew the reduced row echelon form R for the matrix above. What requirement would R have to satisfy to allow you to conclude the list to be dependent? Why? (Note, you do not have to find R or decide if R actually satisfies this requirement.)