EXAM 2

Math 216, 2016-2017 Fall, Clark Bray.

You have 50 minutes.

No notes, no books, no calculators.

YOU MUST SHOW ALL WORK AND EXPLAIN ALL REASONING
TO RECEIVE CREDIT. CLARITY WILL BE CONSIDERED IN GRADING.
All answers must be simplified. All of the policies and guidelines
on the class webpages are in effect on this exam.

Good luck!

Name ________________________________

Disc.: Number__________ TA____________________ Day/Time____________________

"I have adhered to the Duke Community Standard in completing this examination."

Signature: ____________________

1. __________
2. __________
3. __________
4. __________
5. __________
6. __________

Total Score __________ (/100 points)

1. *(20 pts)*

   (a) Use the Wronskian to decide if this list of functions is or is not linearly independent: $f_1(x) = 2x^2 + x + 4,\ f_2(x) = x^2 + 3x + 2,\ f_3(x) = 4x + 7,\ f_4(x) = -x^2 + 2x + 3.$

   (b) Use a dimension argument to draw the same conclusion.

2. *(20 pts)* Find a real fundamental set of solutions to the differential equation $L(y) = 0$ whose characteristic polynomial factors as

$$p(\lambda) = (\lambda - 6)^3(\lambda^3 - 6\lambda^2 + 11\lambda - 6)(\lambda - (4 + 5i))^2(\lambda - (4 - 5i))^2$$

3. *(20 pts)* Find a particular solution to the differential equation below.

$$y' - 4y = xe^{4x}$$

4. *(20 pts)* The input $h(t)$ and output $y(t)$ of a given system are related by the differential equation below.

$$y'' - 2y' + 5y = h(t)$$

Suppose that the input frequency is $f = \omega/2\pi = 2/2\pi$, as represented by the input function $h(t) = \sin(2t)$. Compute the resulting sinusoidal output function, and determine from that output the gain and the phase shift in the system at that frequency.

5. *(10 pts)* Show that $T : C^1[0,1] \to \mathbb{R}$, defined by the formula below, is a linear transformation.

$$T(f) = \int_0^1 e^x f'(x)\,dx$$

6. *(10 pts)*

(a) Compute $(D-3)x^k e^{3x}$, where k is a positive integer.

(b) Compute $(D-3)e^{3x}$.

(c) Use the results above to show that x^2e^{3x} is in the kernel of $L=(D-3)^4(D-2)^3$.