EXAM 2

Math 216, 2015-2016 Spring, Clark Bray.

You have 50 minutes.

No notes, no books, no calculators.

YOU MUST SHOW ALL WORK AND EXPLAIN ALL REASONING
TO RECEIVE CREDIT. CLARITY WILL BE CONSIDERED IN GRADING.
All answers must be simplified. All of the policies and guidelines
on the class webpages are in effect on this exam.

Good luck!

Name ______________________________________

Disc.: Number____________ TA_________________________ Day/Time_________________________

"I have adhered to the Duke Community
Standard in completing this
examination."

Signature: _________________________

1. ____________

2. ____________

3. ____________

4. ____________

5. ____________

Total Score ____________ (/100 points)

1. *(20 pts)* Your friend Bob is wondering if the functions $f(x) = \sin(x)$, $g(x) = \sin(x - \frac{\pi}{4})$, and $h(x) = \sin(x - \frac{\pi}{3})$ are linearly independent.

   (a) Use a dimension argument to give Bob a definitive answer to his question. *(Be sure to be thorough with your explanation.)*

   (b) Use the Wronskian to draw the same conclusion. *(Be sure to be thorough with your explanation.)*

2. *(20 pts)*

(a) Find all of the roots of the polynomial $f(x) = x^3 - x^2 + 8x + 10$.

(b) The differential equation $L(y) = 0$ has characteristic polynomial below. Find a fundamental set of real solutions to this equation.

$$p(\lambda) = \big(\lambda - 3\big)\big(\lambda - 2\big)^3\big(\lambda - (4 - 5i)\big)^2\big(\lambda - (4 + 5i)\big)^2$$

3. *(20 pts)* Find the form (but do not evaluate the coefficients) of a particular solution to the differential equation below, for which the characteristic polynomial is $p(\lambda) = \big(\lambda+1\big)\big(\lambda^2-4\lambda+5\big)$.

$$L(y) = x - 3e^{2x}\cos(x)$$

4. *(20 pts)* Some of the solutions to the differential equation below are products of exponentials and sine waves. What are the two possible frequencies (f, not ω!) for these sine waves?

$$y^{[5]} + 32y = 0$$

5. *(20 pts)* As was discussed in class, let $S = \{\text{solutions to } L(y) = 0\}$, and let $T : S \to \mathbb{R}^n$ be the one-to-one and onto function defined by

$$T(y) = \begin{pmatrix} y(0) \\ y'(0) \\ \vdots \\ y^{[n-1]}(0) \end{pmatrix}$$

Show that T is a linear transformation.