EXAM 1

Math 216, 2015-2016 Fall, Clark Bray.

You have 50 minutes.

No notes, no books, no calculators.

YOU MUST SHOW ALL WORK AND EXPLAIN ALL REASONING
TO RECEIVE CREDIT. CLARITY WILL BE CONSIDERED IN GRADING.
All answers must be simplified. All of the policies and guidelines
on the class webpages are in effect on this exam.

Good luck!

Name ______________________________________

Disc.: Number____________ TA_________________________ Day/Time_________________________

"I have adhered to the Duke Community Standard in completing this examination."

Signature: _________________________

1. ____________
2. ____________
3. ____________
4. ____________
5. ____________
6. ____________
7. ____________

Total Score ____________ (/100 points)

1. *(20 pts)* The augmented matrix $(A|\vec{b}_1)$ row reduces to

$$\left(\begin{array}{ccccc|c} 0 & 1 & 3 & 0 & 2 & 1 \\ 0 & 0 & 0 & 1 & 5 & 3 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

(a) Find the complete set of solutions to the system $A\vec{x} = \vec{b}_1$.

(b) Your friend Bob says that, for a vector $\vec{b}_2$ he has found, the system $A\vec{x} = \vec{b}_2$ has a unique solution. Is there enough information available to decide if Bob is right or wrong? Explain.

(c) The system of equations $A\vec{x} = \vec{b}_3$ is satisfied by $\vec{s} = (2, 1, 5, 3, 1)$. Find the complete set of solutions to the system.

2. *(20 pts)* The systems

$$\left(\begin{array}{c|c} A & \begin{matrix}1\\0\\0\end{matrix}\end{array}\right) \quad \left(\begin{array}{c|c} A & \begin{matrix}0\\1\\0\end{matrix}\end{array}\right) \quad \left(\begin{array}{c|c} A & \begin{matrix}0\\0\\1\end{matrix}\end{array}\right)$$

row reduce to the systems below, respectively:

$$\left(\begin{array}{ccc|c} 1 & 2 & 0 & 3 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 5 \end{array}\right) \quad \left(\begin{array}{ccc|c} 1 & 2 & 0 & 1 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right) \quad \left(\begin{array}{ccc|c} 1 & 2 & 0 & 2 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 4 \end{array}\right)$$

(a) Find a row reduction matrix E for which EA is the reduced row echelon form in the systems above.

(b) Find the complete set of solutions to the system $A\vec{x} = \vec{b}$, with $\vec{b} = (1, 2, 3)$.

(c) Find another matrix $E_2 \neq E$ for which $E_2 A$ is the same reduced row echelon form in the systems above. *(Hint: Think about one additional row operation, that would not change the above reduced row echelon form.)*

3. *(10 pts)* Use Cramer's rule to find an algebraic expression for R in terms of the other variables in this system (it is known that $ac \neq 1$).

$$\begin{pmatrix} 1 & a & 0 \\ 0 & b & 1 \\ c & 1 & 0 \end{pmatrix} \begin{pmatrix} i \\ R \\ K \end{pmatrix} = \begin{pmatrix} L \\ P \\ m \end{pmatrix}$$

4. *(10 pts)* The 3×3 matrix A has rows $\vec{r}_1$, $\vec{r}_2$, $\vec{r}_3$, as indicated below.

$$\begin{pmatrix} - & \vec{r}_1 & - \\ - & \vec{r}_2 & - \\ - & \vec{r}_3 & - \end{pmatrix}$$

The matrix B is obtained from A by the row operation that adds 5 times the third row to the second row.

Use multilinearity explicitly to demonstrate that $\det(B) = \det(A)$.

5. *(10 pts)* Is the list of polynomials below linearly dependent or linearly independent? Be sure to show all of the steps in your reasoning.

$$\{x^2 + x - 1, 2x + 1, 4x^2 - 3\}$$

6. *(10 pts)* Suppose that $\{\vec{p}, \vec{q}, \vec{r}\}$ is linearly dependent. Show that one of these vectors must be a linear combination of the other two.

7. *(20 pts)* Suppose S is the collection of all differentiable functions f for which $f'(0) = 3f'(1)$. Prove or disprove the assertion that S is a vector space.