EXAM 3

Math 216, 2014-2015 Spring, Clark Bray.

You have 50 minutes.

No notes, no books, no calculators.

YOU MUST SHOW ALL WORK AND EXPLAIN ALL REASONING
TO RECEIVE CREDIT. CLARITY WILL BE CONSIDERED IN GRADING.
All answers must be simplified. All of the policies and guidelines
on the class webpages are in effect on this exam.

Good luck!

Name _____

Disc.: Number_____ TA_____ Day/Time_____

"I have adhered to the Duke Community
Standard in completing this
examination."

Signature: _____

1. _____
2. _____
3. _____
4. _____
5. _____

Total Score _____ (/100 points)

1. *(20 pts)* Compute the inverse of the matrix A below without using a row reduction and without using determinants.

$$A = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & -1 & 0 & 1 \end{pmatrix}$$

2. *(20 pts)* V is the inner product space consisting of vectors in $\mathbb{R}^3$, using the usual vector addition and scalar multilication, but with the inner product defined by

$$\langle \vec{v}, \vec{w} \rangle = (A\vec{v}) \cdot (A\vec{w}) = (A\vec{v})^T (A\vec{w}) \quad \text{where} \quad A = \begin{pmatrix} 2 & 3 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 3 \end{pmatrix}$$

Use the Gram-Schmidt method to find an orthonormal basis for the span of the vectors $(1, 0, 0)$ and $(0, 1, 0)$.

3. *(20 pts)* Your friend Bob is trying to find the matrix M. He knows of the vectors

$$\vec{v}_1 = \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix} \quad \vec{v}_2 = \begin{pmatrix} 2 \\ 0 \\ -2 \end{pmatrix} \quad \vec{v}_3 = \begin{pmatrix} -1 \\ 5 \\ 0 \end{pmatrix}$$

that $\vec{v}_1$ and $\vec{v}_3$ are eigenvectors of M with eigenvalue 3, and also that $M\vec{v}_2 = 3\vec{v}_2 + v_3$.

(a) Find the Jordan form F of M, and a basis that achieves this form (be careful to list the vectors in the order consistent with the Jordan form F).

(b) Find matrices A and B, and choose between the expressions ABA^{-1} and $A^{-1}BA$, so that M is computed by the expression you chose.

4. *(20 pts)* Find a fundamental set of solutions to the system

$$\begin{aligned} y_1' &= -7y_1 + 15y_2 \\ y_2' &= -6y_1 + 12y_2 \end{aligned}$$

5. *(20 pts)* The matrices P and P^{-1} are given below.

$$P = \begin{pmatrix} 1 & 3 & 2 \\ 0 & 1 & 1 \\ 2 & 2 & 1 \end{pmatrix} \quad \text{and} \quad P^{-1} = \begin{pmatrix} -1 & 1 & 1 \\ 2 & -3 & -1 \\ -2 & 4 & 1 \end{pmatrix}$$

We consider the system

$$\vec{y}' = \begin{pmatrix} -1 & 9 & 2 \\ -2 & 7 & 1 \\ 0 & 2 & 3 \end{pmatrix} \vec{y}$$

(a) Use the basis formed by the columns of P to try to decouple the above system. Is the resulting system decoupled completely?

(b) Find the solution to the above equation with initial condition $\vec{y}(0) = (2, 1, 1)$ *(you may use any method from the course).*