EXAM 2

Math 216, 2014-2015 Spring, Clark Bray.

You have 50 minutes.

No notes, no books, no calculators.

YOU MUST SHOW ALL WORK AND EXPLAIN ALL REASONING
TO RECEIVE CREDIT. CLARITY WILL BE CONSIDERED IN GRADING.
All answers must be simplified. All of the policies and guidelines
on the class webpages are in effect on this exam.

Good luck!

Name ______________________________

Disc.: Number__________ TA____________________ Day/Time____________________

"I have adhered to the Duke Community Standard in completing this examination."

Signature: ____________________

1. __________

2. __________

3. __________

4. __________

5. __________

6. __________

Total Score __________ (/100 points)

1. *(15 pts)* The columns of the 5×4 matrix A are linearly independent, and B is an invertible 5×5 matrix. Show that the columns of BA are linearly independent. *(Hint: Consider the contrapositive.)*

2. *(20 pts)* Find a particular solution to the differential equation below. *(Hint: Consider a related complex equation.)*

$$y'' + 3y = x \cos x$$

3. *(15 pts)*

(a) The functions

$$\begin{aligned} y_1 &= 13e^{2x}\cos(3x) + 45e^{2x}\sin(3x) + 32e^{5x} \\ y_2 &= 25e^{2x}\cos(3x) + 31e^{2x}\sin(3x) + 64e^{5x} \\ y_3 &= 82e^{2x}\cos(3x) + 43e^{2x}\sin(3x) + 77e^{5x} \end{aligned}$$

form a linearly independent list, and each is a solution to the real constant coefficient differential equation below.

$$a_3y''' + a_2y'' + a_1y' + a_0y = 0$$

What fundamental set of real solutions to this differential equation is the *easiest* to justify given only the information above? Explain *completely* how you know this is in fact a fundamental set of solutions.

(b) Find a fundamental set of solutions to the constant coefficient linear differential equation $L(y) = 0$ whose characteristic polynomial factors as

$$p(\lambda) = (\lambda^2 + 5)(\lambda + 4)^3(\lambda^2 + 2\lambda + 10)$$

4. *(15 pts)* Your grandfather's old car has a suspension system of springs and "shock absorbers" (which provide a friction-like resistance), whose behavior can be related to a mass on a spring in a resistive medium in that it is described by the same sort of differential equation. For your grandfather's car, the modeling differential equation is

$$y''(t) + Ay'(t) + Ky(t) = 0$$

where A is proportional to the resistance of the shock absorbers and K is proportional to the strength of the springs.

The current shock absorbers on his car have $A = 2$. You have also measured that when the car hits a bump in the road the frequency ($f = \frac{\omega}{2\pi}$) of the resulting (decaying) oscillation of the car is $\frac{3}{2\pi}$.

Your grandfather wants to replace his shock absorbers with new ones such that A remains as low as possible (to keep the ride of the car soft and comfortable) while still completely eliminating the oscillation (using the same springs). What is the ideal value of A you should recommend for him to use?

5. *(15 pts)* Find a basis for the kernel of the linear transformation $(D^2 - 6D + 9) : C^\infty \to C^\infty$.

6. *(20 pts)*

(a) What does it mean to say that the function $f : X \to Y$ is a linear transformation? (Be thorough!)

(b) Suppose that $P : A \to B$ and $Q : B \to C$ are linear transformations. Show that the product QP is a linear transformation.