EXAM 3

Math 216, 2014-2015 Fall, Clark Bray.

You have 50 minutes.

No notes, no books, no calculators.

YOU MUST SHOW ALL WORK AND EXPLAIN ALL REASONING
TO RECEIVE CREDIT. CLARITY WILL BE CONSIDERED IN GRADING.
All answers must be simplified. All of the policies and guidelines
on the class webpages are in effect on this exam.

Good luck!

Name ______________________________

Disc.: Number__________ TA____________________ Day/Time____________________

"I have adhered to the Duke Community Standard in completing this examination."

Signature: ____________________

1. __________

2. __________

3. __________

4. __________

5. __________

Total Score __________ (/100 points)

1. *(20 pts)* Suppose that $\vec{v}$ is a complex eigenvector of the real matrix A, with complex eigenvalue $2 - 3i$. Find another eigenvector of A, and find its eigenvalue.

2. *(20 pts)* The matrix A is known to satisfy the following equation.

$$\begin{pmatrix} 3 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 6 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 6 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 6 \end{pmatrix} = \begin{pmatrix} 1 & 7 & 1 & 7 & 2 & 1 & 4 \\ 2 & 6 & 3 & 2 & 1 & 1 & 2 \\ 3 & 5 & 2 & 3 & 3 & 2 & 5 \\ 4 & 4 & 6 & 1 & 3 & 6 & 8 \\ 5 & 3 & 7 & 9 & 3 & 5 & 2 \\ 6 & 2 & 4 & 4 & 6 & 4 & 1 \\ 7 & 1 & 1 & 3 & 7 & 2 & 0 \end{pmatrix}^{-1} \begin{pmatrix} & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & A & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \end{pmatrix} \begin{pmatrix} 1 & 7 & 1 & 7 & 2 & 1 & 4 \\ 2 & 6 & 3 & 2 & 1 & 1 & 2 \\ 3 & 5 & 2 & 3 & 3 & 2 & 5 \\ 4 & 4 & 6 & 1 & 3 & 6 & 8 \\ 5 & 3 & 7 & 9 & 3 & 5 & 2 \\ 6 & 2 & 4 & 4 & 6 & 4 & 1 \\ 7 & 1 & 1 & 3 & 7 & 2 & 0 \end{pmatrix}$$

(a) Compute the product of the matrix A and the vector $\vec{v}_2 = (7, 6, 5, 4, 3, 2, 1)$.

(b) Find all of the eigenvectors of A.

(c) Your friend Bob says he thinks $(12, 1, 1, 1, 4, 5, 6)$ is an eigenvector of A. Given only the information above, can it be determined if Bob is right or wrong, or is there not enough information given? Explain.

3. *(20 pts)* Recall that the angle addition formulas

$$\begin{aligned}\cos(a+b) &= \cos a \cos b - \sin a \sin b \\ \sin(a+b) &= \sin a \cos b + \cos a \sin b\end{aligned}$$

can be used to show $\cos a \cos b = \frac{1}{2}\cos(a+b) + \frac{1}{2}\cos(a-b)$, and similar results.

(a) Show that, when $n \neq m$ are positive integers, the functions $\cos nx$ and $\cos mx$ are orthogonal by the L^2 inner product on the interval $[0, 2\pi]$.

(b) Show that $\sin nx$ and $\cos mx$ are orthogonal using the same inner product.

4. *(20 pts)* The matrix A is known to satisfy the following equation.

$$\begin{pmatrix} & A & \end{pmatrix} = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 6 \\ 0 & 0 & 2 \end{pmatrix}^{-1} \begin{pmatrix} 4 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{pmatrix} \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 6 \\ 0 & 0 & 2 \end{pmatrix}$$

Use the given information to find a fundamental set of solutions for the system $\vec{y}' = A\vec{y}$.

5. *(20 pts)* Find a particular solution to the system of equations below.

$$\begin{aligned} y_1' &= 2y_1 + 3y_2 + 2x^2 \\ y_2' &= 3y_1 + 5y_2 + 3x - 1 \end{aligned}$$