EXAM 2

Math 216, 2014-2015 Fall, Clark Bray.

You have 50 minutes.

No notes, no books, no calculators.

YOU MUST SHOW ALL WORK AND EXPLAIN ALL REASONING
TO RECEIVE CREDIT. CLARITY WILL BE CONSIDERED IN GRADING.
All answers must be simplified. All of the policies and guidelines
on the class webpages are in effect on this exam.

Good luck!

Name ____________________________________

Disc.: Number____________ TA________________________ Day/Time________________________

"I have adhered to the Duke Community Standard in completing this examination."

Signature: ________________________

1. ____________

2. ____________

3. ____________

4. ____________

5. ____________

Total Score ____________ (/100 points)

1. *(20 pts)* The differential equation $L(y) = 0$ has characteristic polynomial

$$p(\lambda) = (\lambda^2 + 3\lambda + 2)(\lambda^2 - 2\lambda + 2)^2$$

Find a fundamental set of real solutions to this differential equation.

2. *(20 pts)* The function $T : C^0 \to \mathbb{R}$ is computed by

$$T(f) = \int_0^1 f(x)e^x \, dx$$

Prove or disprove: T is a linear transformation.

3. *(20 pts)* Recall that P_2 is the vector space of polynomials of degree at most 2. Use the Wronskian as part of your direct demonstration that the dimension of P_2 is 3.

4. *(20 pts)*

(a) Apply the linear transformation $(D-3)$ directly to find a general formula for $(D-3)(x^k e^{3x})$, where k is a positive integer.

(b) Use the above result to show directly that $y = x^4 e^{3x}$ is a solution to $L(y) = 0$, whose characteristic polynomial is

$$p(\lambda) = (\lambda - 4)^3(\lambda - 3)^5(\lambda^2 + 2\lambda - 1)^3$$

5. *(20 pts)*

(a) Find a particular solution to the differential equation

$$z''(t) + z'(t) - z(t) = 7e^{2it}$$

(b) Find solutions to each of the differential equations below.

$$\begin{aligned} u''(t) + u'(t) - u(t) &= 7\cos(2t) \\ v''(t) + v'(t) - v(t) &= 7\sin(2t) \end{aligned}$$