

EXAM 1

Math 216, 2014-2015 Fall, Clark Bray.

You have 50 minutes.

No notes, no books, no calculators.

YOU MUST SHOW ALL WORK AND EXPLAIN ALL REASONING
TO RECEIVE CREDIT. CLARITY WILL BE CONSIDERED IN GRADING.

All answers must be simplified. All of the policies and guidelines
on the class webpages are in effect on this exam.

Good luck!

Name _____

Disc.: Number _____ TA _____ Day/Time _____

“I have adhered to the Duke Community
Standard in completing this
examination.”

1. _____

2. _____

3. _____

4. _____

5. _____

6. _____

Signature: _____

Total Score _____ (/100 points)

1. (24 pts) The matrix $[A|I]$ is row equivalent to

$$\left(\begin{array}{cccc|ccc} 1 & 0 & 2 & 1 & 1 & 2 & 4 \\ 2 & 1 & 3 & 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 0 & 2 & 3 & 1 \end{array} \right)$$

(a) Find the reduced row echelon form (R) of A , and the row reduction matrix (E) that reduces A to R .

(b) Decide if $A\vec{x} = \vec{b}$ has a solution and how many solutions there are, when $\vec{b} = (2, -1, -1)$; and likewise when $\vec{b} = (1, 0, 0)$.

(c) Your friend Bob says that the determinant of E is 5, and thus that the determinant of A must be $1/5$. How would you respond to Bob most helpfully?

2. (16 pts) Find the matrix B such that the rows of BA are related to the rows of A as indicated below.

$$\begin{pmatrix} B \end{pmatrix} \begin{pmatrix} - & A_1 & - \\ - & A_2 & - \\ - & A_3 & - \end{pmatrix} = \begin{pmatrix} - & 3A_3 - A_2 & - \\ - & 2A_1 + A_3 & - \\ - & A_1 - A_2 & - \end{pmatrix}$$

3. (20 pts) The square matrix A has the feature that its three first rows A_1, A_2, A_3 are related by

$$2A_1 - 7A_2 + 5A_3 = \vec{0}$$

Show that the third column of A , and the third column in the cofactor matrix of A , are perpendicular (both being viewed as vectors in \mathbb{R}^n).

4. (20 pts) Decide if the columns of the product matrix B below are linearly independent or linearly dependent.

$$B = \left(\begin{array}{c|c|c} | & | & | \\ \vec{b}_1 & \vec{b}_2 & \vec{b}_3 \\ | & | & | \end{array} \right) = \begin{pmatrix} e & 12 & 2 \\ \pi & 234 & 3\pi \\ \sqrt{134} & \sqrt[3]{21} & e^2 \end{pmatrix} \begin{pmatrix} 2 & 10 & 12 \\ 1 & 12 & 13 \\ 3 & 14 & 17 \end{pmatrix}$$

5. (20 pts) In this question we consider the set W of all real-valued functions on \mathbb{R} for which $|f(0)| \leq |f(1)|$.

(a) Is W closed under addition? Prove or find a counterexample.

(b) Is W closed under scalar multiplication? Prove or find a counterexample.