EXAM 1

Math 216, 2013-2014 Fall, Clark Bray.

You have 50 minutes.

No notes, no books, no calculators.

YOU MUST SHOW ALL WORK AND EXPLAIN ALL REASONING
TO RECEIVE CREDIT. CLARITY WILL BE CONSIDERED IN GRADING.
All answers must be simplified. All of the policies and guidelines
on the class webpages are in effect on this exam.

Good luck!

Name ______________________________

Disc.: Number__________ TA____________________ Day/Time____________________

"I have adhered to the Duke Community Standard in completing this examination."

Signature: ____________________

1. __________

2. __________

3. __________

4. __________

5. __________

6. __________

Total Score __________ (/100 points)

1. *(18 pts)* The 3×3 matrix A can be row reduced to the identity matrix by the following sequence of elementary row operations, performed in the listed order:

Op. 1: 3 times the third row is added to the first row

Op. 2: the first row is multiplied by 2

Op. 3: 5 times the first row is added to the third row

(a) What are the elementary matrices E_1, E_2, E_3 that correspond to these operations?

(b) Compute $\det A$, and A^{-1}, *without computing A itself. (Be sure to show your reasoning.)*

(c) The matrix B row reduces to its reduced row echelon form (below) by the exact same sequence of row operations.

$$\operatorname{rref}(B) = \begin{pmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 5 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Without computing B itself, find a nontrivial relation between the columns $(\vec{b}_1, \vec{b}_2, \vec{b}_3, \vec{b}_4)$ of B. *(Again, be sure to explain your reasoning.)*

2. *(16 pts)* Consider the product R (below) of two upper triangular matrices.

$$\begin{pmatrix} a_1 & a_2 & a_3 \\ 0 & b_2 & b_3 \\ 0 & 0 & c_3 \end{pmatrix} \begin{pmatrix} d_1 & d_2 & d_3 \\ 0 & e_2 & e_3 \\ 0 & 0 & f_3 \end{pmatrix} = \begin{pmatrix} \vec{r}_1 \\ \vec{r}_2 \\ \vec{r}_3 \end{pmatrix} = R$$

(a) What are the coefficients in the equation below giving $\vec{r}_2$ as a linear combination of the row vectors $\vec{d} = (d_1 \quad d_2 \quad d_3)$, $\vec{e} = (0 \quad e_2 \quad e_3)$, and $\vec{f} = (0 \quad 0 \quad f_3)$?

$$\vec{r}_2 = \underline{\qquad}\vec{d} + \underline{\qquad}\vec{e} + \underline{\qquad}\vec{f}$$

(b) Argue from part (a) to conclude the first entry in $\vec{r}_2$.

(c) Use an argument similar to that in parts (a) and (b) to conclude the first and second entries of $\vec{r}_3$.

(d) Find an expression giving the determinant of R.

3. *(16 pts)* Use the permutation definition of determinant to compute the determinant of the matrix below.

$$M = \begin{pmatrix} 1 & 0 & 2 \\ 4 & 2 & 4 \\ 3 & 1 & 5 \end{pmatrix}$$

4. *(17 pts)* Determine if $\{p_1, p_2, p_3\}$ is linearly independent or linearly dependent in the vector space of all polynomials.

$$p_1 = 3x^3 + x^2 + 3x \qquad p_2 = 2x^3 + 2x \qquad p_3 = x^3 - 2x^2 - 7x$$

5. *(17 pts)* Consider the set $V = \{f \in C^1 | f'(x) - x^2 f(x) = 0 \text{ for all } x\}$. Show that V is a vector space.

6. *(16 pts)* The vectors $\vec{v}_1$ and $\vec{v}_2$ are in $\mathbb{R}^3$, and the pair $\{\vec{v}_1, \vec{v}_2\}$ is known to be independent.

(a) Show that this pair of vectors cannot span $\mathbb{R}^3$.

(b) Suppose that $\vec{v}_3$ is not a linear combination of $\vec{v}_1$ and $\vec{v}_2$. Prove that $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ must be a basis for $\mathbb{R}^3$.