EXAM 3

Math 216, 2012-2013 Spring, Clark Bray.

You have 50 minutes.

No notes, no books, no calculators.

YOU MUST SHOW ALL WORK AND EXPLAIN ALL REASONING
TO RECEIVE CREDIT. CLARITY WILL BE CONSIDERED IN GRADING.
All answers must be simplified. All of the policies and guidelines
on the class webpages are in effect on this exam.

Good luck!

Name _____

Disc.:   Number_____ TA_____ Day/Time_____

"I have adhered to the Duke Community
Standard in completing this
examination."

1. _____

2. _____   Signature: _____

3. _____

4. _____

5. _____

6. _____

Total Score _____ (/100 points)

1. *(15 pts)* The matrices A and P are given below.

$$A = \begin{pmatrix} 14 & -4 & -16 \\ -3 & 3 & 4 \\ 9 & -3 & -10 \end{pmatrix} \quad \text{and} \quad P = \begin{pmatrix} 3 & 2 & -4 \\ 1 & 2 & 1 \\ 2 & 1 & -3 \end{pmatrix}$$

(a) Show that the columns of P are eigenvectors of A.

(b) Show that the columns of P form a basis for $\mathbb{R}^3$.

(c) Use the information from the previous two parts to compute $P^{-1}AP$ *without* multiplying the matrices directly. Explain your reasoning.

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2. *(15 pts)* In this problem we consider the inner product space spanned by the functions $\{1, x, x^2\}$, using the L^2 inner product on the interval $[0, 1]$. Compute the angle in this inner product space between the vectors $f_1 = 1 + 2x$ and $f_2 = x^2$.

3. *(15 pts)* In this problem we consider the following arithmetic relating to the matrix A.

$$A = \begin{pmatrix} 71/49 & 24/49 & 30/49 \\ 24/49 & 93/49 & 6/49 \\ 30/49 & 6/49 & 130/49 \end{pmatrix} = \begin{pmatrix} 2/7 & 3/7 & -6/7 \\ 6/7 & 2/7 & 3/7 \\ -3/7 & 6/7 & 2/7 \end{pmatrix} \begin{pmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 2/7 & 6/7 & -3/7 \\ 3/7 & 2/7 & 6/7 \\ -6/7 & 3/7 & 2/7 \end{pmatrix}$$

(a) This equation represents a particular kind of diagonalizability of the matrix A. What kind of diagonalizability is it, what feature of which other matrix in this equation motivates this terminology, and what feature of A ensures that it will have this kind of diagonalizability?

(b) Interpreting A as $[T]_{\mathcal{S}}^{\mathcal{S}}$, find (standard basis representations of vectors in) a basis $\mathcal{V}$ such that $[T]_{\mathcal{V}}^{\mathcal{V}}$ is diagonal.

4. *(20 pts)* Find a fundamental set of solutions to the system

$$
\begin{aligned}
y_1' &= 3y_1 - 4y_2 \\
y_2' &= 4y_1 + 3y_2
\end{aligned}
$$

5. *(15 pts)* Suppose that $A = [T]^{\mathcal{S}}_{\mathcal{S}}$, and

$$[T]^{\mathcal{V}}_{\mathcal{V}} = \begin{pmatrix} 5 & 0 & 0 & 0 \\ 0 & 5 & 1 & 0 \\ 0 & 0 & 5 & 1 \\ 0 & 0 & 0 & 5 \end{pmatrix} \quad \text{where} \quad \mathcal{V} = \left\{ \begin{pmatrix} 1 \\ 0 \\ 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 5 \\ 6 \end{pmatrix}, \begin{pmatrix} 2 \\ 3 \\ 8 \\ 4 \end{pmatrix}, \begin{pmatrix} -3 \\ -2 \\ 0 \\ 1 \end{pmatrix} \right\}$$

Find a fundamental set of solutions to the system $\vec{y}' = A\vec{y}$.

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6. *(20 pts)* The matrix A below has characteristic polynomial $p(\lambda) = (\lambda - 3)(\lambda - 2)^2$, and the two vectors $\vec{v}$ and $\vec{w}$ are known eigenvectors. Find a third vector $\vec{u}$ which, combined with $\vec{v}$ and $\vec{w}$, forms a Jordan basis for A.

$$A = \begin{pmatrix} 11 & -3 & -2 \\ 2 & 2 & 0 \\ 33 & -12 & -6 \end{pmatrix} \qquad \vec{v} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} \qquad \vec{w} = \begin{pmatrix} 0 \\ 2 \\ -3 \end{pmatrix}$$