EXAM 3

Math 216, 2012-2013 Fall, Clark Bray.

You have 50 minutes.

No notes, no books, no calculators.

YOU MUST SHOW ALL WORK AND EXPLAIN ALL REASONING
TO RECEIVE CREDIT. CLARITY WILL BE CONSIDERED IN GRADING.
All answers must be simplified. All of the policies and guidelines
on the class webpages are in effect on this exam.

Good luck!

Name ______________________________

Disc.: Number__________ TA____________________ Day/Time____________________

"I have adhered to the Duke Community Standard in completing this examination."

Signature: ____________________

1. __________
2. __________
3. __________
4. __________
5. __________
6. __________
7. __________

Total Score __________ (/100 points)

1. *(10 pts)* Find an eigenvector with eigenvalue 2 for the matrix below.

$$\begin{pmatrix} 5 & 0 & -1 \\ 1 & 3 & -1 \\ 1 & -2 & 3 \end{pmatrix}$$

2. *(15 pts)* Show that if a matrix A is real and symmetric, and if $\vec{v}$ is a nonzero eigenvector with eigenvalue λ, then λ must be real. *(Hint: Consider $\langle A\vec{v}, \vec{v}\rangle_H$.)*

3. *(15 pts)* Use the idea of diagonalization to find a matrix B for which the vectors below are eigenvectors with eigenvalues 3 and 2 (respectively).

$$\vec{v}_1 = \begin{bmatrix} 3 \\ 4 \end{bmatrix} \quad \text{and} \quad \vec{v}_1 = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

4. *(15 pts)* Find the Jordan canonical form and a Jordan basis for the matrix A below. You may use the fact that the characteristic polynomial is $p(\lambda) = (\lambda - 2)^3$, and that the two augmented matrices listed below are row equivalent as indicated.

$$A = \begin{pmatrix} 9 & 2 & -7 \\ 16 & 6 & -16 \\ 11 & 3 & -9 \end{pmatrix}$$

$$\left(\begin{array}{ccc|ccc} 7 & 2 & -7 & 1 & 0 & 0 \\ 16 & 4 & -16 & 0 & 1 & 0 \\ 11 & 3 & -11 & 0 & 0 & 1 \end{array}\right) \text{ is row equivalent to } \left(\begin{array}{ccc|ccc} 1 & 0 & -1 & 0 & 3/4 & -1 \\ 0 & 1 & 0 & 0 & -11/4 & 4 \\ 0 & 0 & 0 & 1 & 1/4 & -1 \end{array}\right)$$

5. *(15 pts)* Complete the Gram-Schmidt orthonormalization of the basis $\mathcal{V} = \{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ of a subspace of $\mathbb{R}^4$ (using the usual dot product), noting that the first two vectors in this list are already orthonormal.

$$\vec{v}_1 = \begin{bmatrix} 2/7 \\ 3/7 \\ 0 \\ 6/7 \end{bmatrix} \qquad \vec{v}_2 = \begin{bmatrix} -6/7 \\ 0 \\ 3/7 \\ 2/7 \end{bmatrix} \qquad \vec{v}_3 = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$$

6. *(15 pts)* Suppose that for a matrix B, we have three eigenvectors (below) with eigenvalues 3, -2, and 4 (respectively).

$$\vec{v}_1 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \quad \text{and} \quad \vec{v}_2 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \quad \text{and} \quad \vec{v}_3 = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$

(a) Find a fundamental set of solutions for the equation $\vec{y}' = B\vec{y}$.

(b) Compute the matrix e^{xB} *(Hint: Recall that the i^{th} column is $e^{xB}\vec{e}_i$, and note that each $\vec{e}_i$ is an easy linear combination of the eigenvectors.)*

(c) Solve the initial value problem $\vec{y}' = B\vec{y}$, with $\vec{y}(0) = (2, 5, 7)$.

7. *(15 pts)* Use the arithmetic indicated below to find a fundamental set of solutions to the system $\vec{y}' = A\vec{y}$. *(Hint: You may find it useful to note that two of the matrices expressed below are both transposes and inverses of each other.)*

$$A = \begin{pmatrix} 86/49 & 13/49 & 18/49 \\ 6/49 & 116/49 & 40/49 \\ -45/49 & 12/49 & 92/49 \end{pmatrix} = \begin{pmatrix} 2/7 & 3/7 & -6/7 \\ 6/7 & 2/7 & 3/7 \\ -3/7 & 6/7 & 2/7 \end{pmatrix} \begin{pmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{pmatrix} \begin{pmatrix} 2/7 & 6/7 & -3/7 \\ 3/7 & 2/7 & 6/7 \\ -6/7 & 3/7 & 2/7 \end{pmatrix}$$