EXAM 2

Math 216, 2012-2013 Fall, Clark Bray.

You have 50 minutes.

No notes, no books, no calculators.

YOU MUST SHOW ALL WORK AND EXPLAIN ALL REASONING
TO RECEIVE CREDIT. CLARITY WILL BE CONSIDERED IN GRADING.
All answers must be simplified. All of the policies and guidelines
on the class webpages are in effect on this exam.

Good luck!

Name _____

Disc.: Number_____ TA_____ Day/Time_____

1. _____
2. _____
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7. _____
8. _____
9. _____

"I have adhered to the Duke Community Standard in completing this examination."

Signature: _____

Total Score _____ (/100 points)

1. *(6 pts)* Can the Wronskian alone (without using other knowledge of these functions) allow us to conclude whether the collection $\{x^2, 2x+5, 3x-7, 2x^2+1\}$ is independent or dependent?

2. *(6 pts)* By trial and error, Bob has found three valid solutions to the equation $2y'''-3y''+2y'+y=0$, and he has correctly computed that the Wronskian is the zero function. Using this information, can you determine if his three functions form an independent set or a dependent set? Explain.

3. *(16 pts)* Find a real fundamental set of solutions to the differential equation

$$y''' + 2y'' + 4y' + 8y = 0$$

4. *(9 pts)* Write down the form of the particular solution to the differential equation below (but do NOT solve for the constants).

$$y''' + 2y'' + 4y' + 8y = x^5 e^{-2x}$$

5. *(15 pts)* Find the resonant particular solution to the differential equation below.

$$y'' + \omega_0^2 y = \sin \omega_0 t$$

6. *(11 pts)* The function $\delta_a^{[n]} : C^\infty \to \mathbb{R}$ is defined by $\delta_c^{[n]}(f) = f^{[n]}(c)$. Show that this is a linear transformation.

7. *(11 pts)* Suppose that S and T are linear transformations from a vector space V to a vector space W. Use the definitions of the operations on linear transformations to show that $c(S+T) = cS+cT$.

8. *(11 pts)* Let $D : C^\infty \to C^\infty$ be defined by $D(f) = f'$. Without citing a previous result, compute $(D-3)^4(x^7e^{3x})$.

9. *(15 pts)* Let $f_1 = \sin 2x$, $f_2 = \cos 2x$, $f_3 = 3\cos(2x - \frac{\pi}{3})$, and let V be the vector space with basis $\beta = \{f_1, f_2\}$. The linear transformation $L : V \to V$ is defined by $L(y) = y'' - 2y' + 3y$. Compute the items listed below. *(Hint: Recall that* $\cos(a + b) = \cos a \cos b - \sin a \sin b$*.)*

   (a) $[f_3]_\beta$

   (b) $[L]_\beta^\beta$

   (c) $[L(f_3)]_\beta$ (without computing $L(f_3)$ directly)