EXAM 1

Math 216, 2012-2013 Fall, Clark Bray.

You have 50 minutes.

No notes, no books, no calculators.

YOU MUST SHOW ALL WORK AND EXPLAIN ALL REASONING
TO RECEIVE CREDIT. CLARITY WILL BE CONSIDERED IN GRADING.
All answers must be simplified. All of the policies and guidelines
on the class webpages are in effect on this exam.

Good luck!

Name ______________________________

Disc.: Number__________ TA____________________ Day/Time____________________

1. __________
2. __________
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5. __________
6. __________
7. __________

"I have adhered to the Duke Community Standard in completing this examination."

Signature: ____________________

Total Score __________ (/100 points)

1. *(16 pts)* The reduced row echelon form of the augmented matrix for the system $A\vec{x} = \vec{b}$ (where $\vec{x} = (x_1, x_2, x_3, x_4)$), the product E of the elementary matrices used in that row reduction, and E^{-1}, are given below.

$$\text{rref}(A|\vec{b}) = \left(\begin{array}{cccc|c} 1 & 0 & 3 & 0 & 2 \\ 0 & 1 & 1 & 0 & 5 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right) \quad \text{and} \quad E = \begin{pmatrix} 2 & 3 & 2 \\ 4 & -1 & 0 \\ 2 & 1 & 1 \end{pmatrix} \quad \text{and} \quad E^{-1} = \begin{pmatrix} 0.5 & 0.5 & -1 \\ 2 & 1 & -4 \\ -3 & -2 & 7 \end{pmatrix}$$

(a) Find the complete set of solutions to the system $A\vec{x} = \vec{b}$.

(b) Find a vector $\vec{c}$ for which $A\vec{x} = \vec{c}$ has no solutions.

(c) Find the original matrix A.

(d) Find bases for the row space, column space, and null space of A.

2. *(14 pts)* Use a SINGLE row reduction (you can row reduce anything you choose) to find the reduced row echelon form, determinant, and inverse (or show it does not exist) of the matrix A below. (You must make clear use of the single row reduction in all three computations.)

$$A = \begin{pmatrix} 2 & 4 \\ 1 & 7 \end{pmatrix}$$

3. *(14 pts)* Consider the related matrices below.

$$M = \begin{pmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{pmatrix} \quad \text{and}$$

$$M_1 = \begin{pmatrix} m_{11} & m_{12} & m_{13} \\ 1 & 0 & 0 \\ m_{31} & m_{32} & m_{33} \end{pmatrix} \quad M_2 = \begin{pmatrix} m_{11} & m_{12} & m_{13} \\ 0 & 1 & 0 \\ m_{31} & m_{32} & m_{33} \end{pmatrix} \quad M_3 = \begin{pmatrix} m_{11} & m_{12} & m_{13} \\ 0 & 0 & 1 \\ m_{31} & m_{32} & m_{33} \end{pmatrix}$$

Suppose also that $\det(M_1) = 3$, $\det(M_2) = 18$, $\det(M_3) = -5$.

(a) Find the simplest possible formula for the determinant of the above matrix M in terms of its entries $\{m_{ij}\}$.

(b) Find a similar formula for the determinant of the matrix N given by

$$N = \begin{pmatrix} m_{31} & m_{32} & m_{33} \\ m_{21} & m_{22} & m_{23} \\ m_{11} & m_{12} & m_{13} \end{pmatrix}$$

4. *(14 pts)* The matrix A below can be written as the product $A = M_1 M_2$ of two elementary matrices, M_1 and M_2. Find these elementary matrices M_1 and M_2.

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 7 \end{pmatrix}$$

5. *(14 pts)* Consider the vectors below in the vector space P_2 of polynomials of degree at most 2. (Do not use here previous knowledge you might have about the dimension of P_2.)

$$\begin{aligned} p_1(x) &= x^2 - 2x \\ p_2(x) &= x^2 + 2x \\ p_3(x) &= x + 1 \\ p_4(x) &= 8x - 5 \end{aligned}$$

Show that $\beta = \{p_1, p_2, p_3\}$ is a basis for P_2, and compute $[p_4]_\beta$.

6. *(14 pts)* Your friend Bob wishes to form a vector space consisting of all invertible 2×2 matrices, using the operations below.

$$\begin{aligned} A \oplus B &= AB \\ c \odot A &= cA \end{aligned}$$

Does this form a vector space? If so, prove it; if not, identify one requirement that fails, and show this failure with an explicit example.

7. *(14 pts)* Let W be the subset of C^∞ consisting of all function of the form Ae^{x+B}, where A and B can be any real numbers.

   Your friend Bob says that W can be viewed as a subspace of C^∞; is he right? If he is, what is the dimension of W?