EXAM 3

Math 107, 2011-2012 Spring, Clark Bray.

You have 50 minutes.

No notes, no books, no calculators.

YOU MUST SHOW ALL WORK AND EXPLAIN ALL REASONING
TO RECEIVE CREDIT. CLARITY WILL BE CONSIDERED IN GRADING.
All answers must be simplified. All of the policies and guidelines
on the class webpages are in effect on this exam.

Good luck!

Name ______________________________

Rec: Number__________ TA____________________ Day/Time____________________

"I have adhered to the Duke Community
Standard in completing this
examination."

Signature: ____________________

1. __________

2. __________

3. __________

4. __________

5. __________

Total Score __________ (/100 points)

1. *(20 pts)* Note the following arithmetic:

$$\begin{pmatrix} 2 & 0 & 6 \\ 4 & -3 & -2 \\ -4 & 3 & 0 \end{pmatrix} = \underbrace{\begin{pmatrix} 2 & 12 & 12 \\ -2 & -8 & -11 \\ 2 & 6 & 9 \end{pmatrix}}_{A} \begin{pmatrix} 1 & 0 & -3 \\ 2 & -1 & 1 \\ -2 & 1 & 0 \end{pmatrix}$$

(a) Find ALL of the eigenvectors and eigenvalues of the matrix A.

(b) Find the characteristic polynomial of A, WITHOUT computing a determinant.

(c) Viewing A as representing a linear transformation T with respect to the standard basis $\mathcal{S}$, find another basis $\mathcal{V}$ and a diagonal matrix D that represents the same linear transformation T with respect to $\mathcal{V}$.

2. *(20 pts)* Compute the coefficients c_1, c_2, c_3 in the equation below WITHOUT performing a row reduction or matrix inversion. (Be sure to explain your reasoning!)

$$\begin{pmatrix} 5 \\ 2 \\ 1 \end{pmatrix} = c_1 \begin{pmatrix} -6/7 \\ 2/7 \\ 3/7 \end{pmatrix} + c_2 \begin{pmatrix} 2/7 \\ -3/7 \\ 6/7 \end{pmatrix} + c_3 \begin{pmatrix} 3/7 \\ 6/7 \\ 2/7 \end{pmatrix}$$

3. *(20 pts)* Show that the Hermitian dot product, defined by

$$\langle \vec{v}, \vec{w} \rangle_H = \sum v_i \overline{w_i} = \vec{v}^T \overline{\vec{w}}$$

has the property that

$$\langle \vec{v}, \vec{v} \rangle_H \geq 0, \text{ with equality iff } \vec{v} = \vec{0}$$

4. *(20 pts)* The matrix A has Jordan form

$$J = \begin{pmatrix} 5 & 1 & 0 & 0 \\ 0 & 5 & 0 & 0 \\ 0 & 0 & 5 & 1 \\ 0 & 0 & 0 & 5 \end{pmatrix}$$

and corresponding Jordan basis $\{\vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4\}$, with

$$\vec{v}_1 = \begin{bmatrix} 2 \\ 4 \\ 3 \\ 1 \end{bmatrix} \qquad \vec{v}_2 = \begin{bmatrix} 3 \\ 5 \\ 1 \\ 7 \end{bmatrix} \qquad \vec{v}_3 = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 2 \end{bmatrix} \qquad \vec{v}_4 = \begin{bmatrix} 2 \\ 1 \\ 5 \\ 6 \end{bmatrix}$$

Find a fundamental set of solutions to the system of equations $\vec{y}\,' = A\vec{y}$.

5. *(20 pts)* Find a Jordan basis and the Jordan canonical form for the matrix

$$A = \begin{pmatrix} 15 & 24 & -40 \\ -1 & 3 & 4 \\ 2 & 5 & -3 \end{pmatrix}$$

given that the characteristic polynomial of A is $p(\lambda) = (\lambda - 5)^3$, and

$$\left(\begin{array}{ccc|ccc} 10 & 24 & -40 & 1 & 0 & 0 \\ -1 & -2 & 4 & 0 & 1 & 0 \\ 2 & 5 & -8 & 0 & 0 & 1 \end{array}\right)$$

is row equivalent to

$$\left(\begin{array}{ccc|ccc} 1 & 0 & -4 & 0 & -5 & -2 \\ 0 & 1 & 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 1 & 2 & -4 \end{array}\right)$$