EXAM 2

Math 107, 2011-2012 Spring, Clark Bray.

You have 50 minutes.

No notes, no books, no calculators.

YOU MUST SHOW ALL WORK AND EXPLAIN ALL REASONING
TO RECEIVE CREDIT. CLARITY WILL BE CONSIDERED IN GRADING.
All answers must be simplified. All of the policies and guidelines
on the class webpages are in effect on this exam.

Good luck!

Name _____

Rec: Number_____ TA_____ Day/Time_____

"I have adhered to the Duke Community
Standard in completing this
examination."

1. _____

2. _____ Signature: _____

3. _____

4. _____

5. _____

6. _____

Total Score _____ (/100 points)

1. *(20 pts)* In this problem we consider the vector space P_3 of all polynomials of degree less than or equal to 3. Find a (convenient) basis for P_3, show that it is a basis, and use this conclusion to compute $\dim(P_3)$. (Note, this means that you cannot make any assumptions about the dimension of P_3 while confirming your basis.)

2. *(15 pts)* By non-standard methods, your friend Bob has found four valid solutions to the differential equation

$$y'''' - 8y''' - 4y'' + 5y' + 2y = 0$$

Bob is now interested in the question:

Q: Are these four functions linearly independent or linearly dependent?

He has tried to compute the complete Wronskian, but he found the algebra to be too inconvenient to work out completely. However, he has been able to compute (correctly) that the value of the Wronskian at $x = 1$ is $w(1) = 0$. Based on this, he makes the following assertions:

(a) If $w(1)$ had turned out to be nonzero, that would have proved that the four functions were independent.

(b) The fact that $w(1) = 0$ does not demonstrate that the four functions are independent.

(c) The fact that $w(1) = 0$ does not demonstrate anything else relevant to the question (Q), so further calculations are necessary.

Identify each of the above three assertions as true or false; for each assertion that you identify as false, explain why Bob is wrong.

3. *(20 pts)* Find a fundamental set of real solutions to the differential equation below:

$$y'''' - 4y''' + 27y = 0$$

(Hint: One real solution is $e^{-x}\sin(\sqrt{2}x)$*.)*

4. *(15 pts)* Write down the form (but do not evaluate the constants!) for a particular solution to the differential equation below:

$$y''' - y'' + 3y' + 5y = x^2 e^x \cos(2x)$$

5. *(15 pts)* A mass on a spring in a frictionless medium is moving with position given by

$$u(t) = \cos(168t) - \cos(162t)$$

Use the angle addition formulas below to write this function as a product of sinusoidal waves.

$$\begin{aligned} \cos(a+b) &= \cos(a)\cos(b) - \sin(a)\sin(b) \\ \sin(a+b) &= \sin(a)\cos(b) + \cos(a)\sin(b) \end{aligned}$$

6. *(15 pts)* The linear transformation $D : C^\infty \to C^\infty$ is computed by $D(f) = f'$, the linear transformation $T_a : C^\infty \to C^\infty$ (where a is a constant) is computed by $T_a(f) = af$, and the linear transformation $T_g : C^\infty \to C^\infty$ (where $g(x) = e^x$) is computed by $T_g(f) = gf = e^x f$

   Prove your answer to each of the questions below:

   (a) Do D and T_a commute (that is, does $DT_a = T_a D$)?

   (b) Do D and T_g commute?

   (c) Do T_a and T_g commute?