EXAM 1

Math 107, 2011-2012 Spring, Clark Bray.

You have 50 minutes.

No notes, no books, no calculators.

YOU MUST SHOW ALL WORK AND EXPLAIN ALL REASONING
TO RECEIVE CREDIT. CLARITY WILL BE CONSIDERED IN GRADING.
All answers must be simplified. All of the policies and guidelines
on the class webpages are in effect on this exam.

Good luck!

Name ___________________________________

ID number_______________________

"I have adhered to the Duke Community
Standard in completing this
examination."

Signature: _______________________

1. ___________

2. ___________

3. ___________

4. ___________

5. ___________

6. ___________

Total Score ___________ (/100 points)

1. *(15 pts)* Find the unique matrix A that satisfies the equation below.

$$\begin{pmatrix} & & \\ & A & \\ & & \end{pmatrix} \begin{pmatrix} 3 & 5 & 1 & -1 \\ 2 & 0 & 0 & 5 \\ 7 & 18 & 21 & 2 \end{pmatrix} = \begin{pmatrix} 6 & 10 & 2 & -2 \\ 9 & 18 & 21 & 7 \\ 4 & 0 & 0 & 10 \end{pmatrix}$$

2. *(15 pts)* Suppose your friend Bob tells you that the system $A\vec{x} = \vec{b}$ has at least one solution for every vector $\vec{b}$. Suppose that Bob also tells you he has found two distinct solutions to the system when $\vec{b} = \vec{b}_1$.

   Use observations about the pivots in rref(A) and rref(A^T) to show that A^T cannot have the existence property.

3. *(15 pts)*

(a) Use a row reduction to compute the inverse of the matrix

$$A = \begin{pmatrix} 3 & 8 \\ 1 & 2 \end{pmatrix}$$

(b) Explicitly use the row reduction above to compute the determinant of A.

(c) By any appropriate means, find all values of c for which the matrix below is not invertible.

$$\begin{pmatrix} 3 & c \\ 1 & 2 \end{pmatrix}$$

4. *(20 pts)* The nonsingular 3×3 matrix M is brought to its reduced row echelon form by way of the following ordered sequence of elementary row operations:

   1. The first row is added to the second row.
   2. The second row is multiplied by 7.
   3. Five times the third row is added to the first row.
   4. The second and third rows are switched.

   Write the matrix M as a product of elementary matrices.

5. *(20 pts)* The $n \times n$ matrix A has rows $A_1, \ldots, A_n$. The matrix obtained by crossing out the ith row and jth column of A is M_{ij}.

The matrix P is obtained by replacing the ith row of A by $V = (v_1 \ \cdots \ v_n)$. The matrix Q is obtained by replacing the ith row of A by $W = (w_1 \ \cdots \ w_n)$. The matrix R is obtained by replacing the ith row of A by $aV + bW$:

$$R = \begin{bmatrix} A_1 \\ \vdots \\ aV + bW \\ \vdots \\ A_n \end{bmatrix} \quad P = \begin{bmatrix} A_1 \\ \vdots \\ V \\ \vdots \\ A_n \end{bmatrix} \quad Q = \begin{bmatrix} A_1 \\ \vdots \\ W \\ \vdots \\ A_n \end{bmatrix}$$

Using this notation then, the ith row cofactor expansion for the determinant of P is

$$\det P = \sum_{j=1}^{n} v_j (-1)^{i+j} \det(M_{ij})$$

Prove the multilinearity of determinant in rows by showing that

$$\det R = a \det P + b \det Q$$

6. *(15 pts)* Determine if the collection of vectors below is linearly independent or linearly dependent.

$$\left\{ \begin{bmatrix} 2 \\ 3 \\ 1 \\ -2 \end{bmatrix}, \begin{bmatrix} 5 \\ 1 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 9 \\ 7 \\ 3 \\ -2 \end{bmatrix} \right\}$$