EXAM 3

Math 107, 2010-2011 Spring, Clark Bray.

You have 50 minutes.

No notes, no books, no calculators.

YOU MUST SHOW ALL WORK AND EXPLAIN ALL REASONING
TO RECEIVE CREDIT. CLARITY WILL BE CONSIDERED IN GRADING.
All answers must be simplified. All of the policies and guidelines
on the class webpages are in effect on this exam.

Good luck!

Name ______________________________

ID number____________________

"I have adhered to the Duke Community Standard in completing this examination."

Signature: ____________________

1. __________
2. __________
3. __________
4. __________
5. __________

Total Score __________ (/100 points)

1. *(20 pts)* Find the matrix $A = [T]_{\mathcal{S}}^{\mathcal{S}}$, given the information below:

$$[T]_{\mathcal{V}}^{\mathcal{V}} = \begin{pmatrix} 3 & 1 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

$$\mathcal{V} = \{\vec{v}_1, \vec{v}_2, \vec{v}_3\}, \quad [\vec{v}_1]_{\mathcal{S}} = \begin{bmatrix} 1 \\ 4 \\ -2 \end{bmatrix} \quad [\vec{v}_2]_{\mathcal{S}} = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} \quad [\vec{v}_3]_{\mathcal{S}} = \begin{bmatrix} 2 \\ 5 \\ -2 \end{bmatrix}$$

$$\begin{pmatrix} -9 & 8 & 11 \\ -2 & 2 & 3 \\ 8 & -7 & -10 \end{pmatrix} \begin{pmatrix} 1 & 3 & 2 \\ 4 & 2 & 5 \\ -2 & 1 & -2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

2. *(30 pts)* Find a Jordan basis and the Jordan canonical form for the matrix

$$\begin{pmatrix} 7 & 6 & -7 \\ -5 & -1 & 6 \\ 1 & 3 & 0 \end{pmatrix}$$

(extra space if needed)

3. *(10 pts)* Show by direct calculation that the Hermitian transpose, defined by $A^* = \bar{A}^T$, satisfies the following equation.

$$\langle A\vec{v}, \vec{w}\rangle_H = \langle \vec{v}, A^*\vec{w}\rangle_H$$

4. *(20 pts)* Find a fundamental set of solutions to the system of differential equations described by $\vec{y}' = A\vec{y}$, where

$$A = \begin{pmatrix} 4 & 1 & -3 \\ 43 & -7 & 191 \\ 3 & -1 & 18 \end{pmatrix}$$

Here is some arithmetic that you might find useful:

$$\begin{pmatrix} 4 & 1 & -3 \\ 43 & -7 & 191 \\ 3 & -1 & 18 \end{pmatrix} = \begin{pmatrix} 5 & 2 & 1 \\ 2 & 7 & 3 \\ -1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 5 & 1 & 0 \\ 0 & 5 & 1 \\ 0 & 0 & 5 \end{pmatrix} \begin{pmatrix} 0 & 0 & -1 \\ -3 & 1 & -13 \\ 7 & -2 & 31 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 5 & 2 & 1 \\ 2 & 7 & 3 \\ -1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & -1 \\ -3 & 1 & -13 \\ 7 & -2 & 31 \end{pmatrix}$$

5. *(20 pts)* Find a particular solution to the system of differential equations $\vec{y}' = A\vec{y} + \vec{g}$, with

$$A = \begin{pmatrix} -5 & -3 \\ 2 & 0 \end{pmatrix} \quad \text{and} \quad \vec{g}(x) = \begin{bmatrix} 0 \\ x \end{bmatrix}$$

(In case it might be useful, note that the vectors $(3, -2)$ *and* $(-1, 1)$ *are eigenvectors.)*