EXAM 1
Math 107, 2010-2011 Spring, Clark Bray.

You have 50 minutes.

No notes, no books, no calculators.

YOU MUST SHOW ALL WORK AND EXPLAIN ALL REASONING TO RECEIVE CREDIT. CLARITY WILL BE CONSIDERED IN GRADING. All answers must be simplified. All of the policies and guidelines on the class webpages are in effect on this exam.

Good luck!

Name __________________________________________

ID number __________________________

1. ________  “I have adhered to the Duke Community Standard in completing this examination.”

2. ________

3. ________

4. ________

5. ________

6. ________

7. ________

Signature: __________________________

Total Score ____________ (/100 points)
1. (15 pts) Find the complete set of solutions to the system of equations below.

\[
\begin{align*}
    x_1 + 2x_2 - x_3 + 2x_4 &= 13 \\
    2x_1 + 4x_2 - x_3 + 6x_4 - 2x_5 &= 19 \\
    11x_1 + 22x_2 - 7x_3 + 30x_4 - 11x_5 &= 100 \\
    -3x_1 - 6x_2 + 2x_3 - 8x_4 + 3x_5 &= -27
\end{align*}
\]

You may use the arithmetic fact below, and also that the determinant of the left matrix on the left side of the equation below is -1.

\[
\begin{pmatrix}
    1 & 3 & 0 & 2 \\
    0 & 2 & 1 & 5 \\
    1 & 1 & 0 & 1 \\
    0 & -1 & 1 & 3
\end{pmatrix}
\begin{pmatrix}
    1 & 2 & -1 & 2 & 0 \\
    2 & 4 & -1 & 6 & -2 \\
    11 & 22 & -7 & 30 & -11 \\
    -3 & -6 & 2 & -8 & 3
\end{pmatrix}
= \begin{pmatrix}
    1 & 2 & 0 & 4 & 0 \\
    0 & 0 & 1 & 2 & 0 \\
    0 & 0 & 0 & 0 & 1 \\
    0 & 0 & 0 & 0 & 0
\end{pmatrix}
\]
2. (10 pts) The rows of the 5x5 nonsingular matrices $A$ and $B$ have the following in common:

(a) $B_1$ is $A_1$ plus 2 times $A_2$.
(b) $B_2$ is 5 times $A_5$ minus 6 times $A_3$.
(c) $B_3$ is 8 times $A_2$ minus $A_1$.
(d) $B_4$ is 3 times $A_5$ minus $A_4$.
(e) $B_5$ is $A_4$.

Find the unique matrix $C$ that satisfies $CA = B$. 
3. (15 pts) Find the inverse matrix for the matrix $N$ below.

$$N = \begin{pmatrix} 5 & 2 & 2 \\ 1 & -5 & 0 \\ -3 & 2 & -1 \end{pmatrix}$$
4. (15 pts) Show that the square matrix $A$ is nonsingular ($\text{rref}(A) = I$) if and only if $A$ is invertible.
5. * (15 pts) Use permutations to compute the determinant of the matrix

\[
\begin{pmatrix}
5 & -6 & 10 \\
1 & 3 & -2 \\
2 & 4 & 1
\end{pmatrix}
\]
6. (15 pts) We consider here the matrix $R$ below. Compute the entry in the 3rd row and 4th column of the matrix $R^{-1}$.

$$R = \begin{pmatrix}
2 & 2 & 2 & 3 \\
1 & 1 & 1 & 1 \\
1 & 3 & 2 & 6 \\
4 & -2 & -4 & 1
\end{pmatrix}$$
7. (15 pts) You and your friend Bob are considering the collection \( \{\vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4\} \) of vectors in \( \mathbb{R}^5 \), where

\[
\vec{v}_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 0 \end{bmatrix}, \quad \vec{v}_2 = \begin{bmatrix} 0 \\ 2 \\ 4 \\ 5 \\ 7 \end{bmatrix}, \quad \vec{v}_3 = \begin{bmatrix} 2 \\ 0 \\ 1 \\ 5 \\ 12 \end{bmatrix}, \quad \vec{v}_4 = \begin{bmatrix} 3 \\ -4 \\ -4 \\ 0 \\ 10 \end{bmatrix}
\]

Bob observes, correctly, that none of these four vectors is a nontrivial linear combination of the other vectors. He cites this fact *alone* as demonstrating that this collection of vectors is linearly independent.

Is Bob’s reasoning correct? Explain why or why not. Is Bob’s conclusion correct? Explain why or why not.