EXAM 1

Math 107, 2010-2011 Spring, Clark Bray.

You have 50 minutes.

No notes, no books, no calculators.

YOU MUST SHOW ALL WORK AND EXPLAIN ALL REASONING
TO RECEIVE CREDIT. CLARITY WILL BE CONSIDERED IN GRADING.
All answers must be simplified. All of the policies and guidelines
on the class webpages are in effect on this exam.

Good luck!

Name ______________________________

ID number____________________

"I have adhered to the Duke Community Standard in completing this examination."

Signature: ____________________

1. __________
2. __________
3. __________
4. __________
5. __________
6. __________
7. __________

Total Score __________ (/100 points)

1. *(15 pts)* Find the complete set of solutions to the system of equations below.

$$\begin{aligned} x_1 + 2x_2 - x_3 + 2x_4 &= 13 \\ 2x_1 + 4x_2 - x_3 + 6x_4 - 2x_5 &= 19 \\ 11x_1 + 22x_2 - 7x_3 + 30x_4 - 11x_5 &= 100 \\ -3x_1 - 6x_2 + 2x_3 - 8x_4 + 3x_5 &= -27 \end{aligned}$$

You may use the arithmetic fact below, and also that the determinant of the left matrix on the left side of the equation below is -1.

$$\begin{pmatrix} 1 & 3 & 0 & 2 \\ 0 & 2 & 1 & 5 \\ 1 & 1 & 0 & 1 \\ 0 & -1 & 1 & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 & -1 & 2 & 0 \\ 2 & 4 & -1 & 6 & -2 \\ 11 & 22 & -7 & 30 & -11 \\ -3 & -6 & 2 & -8 & 3 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 0 & 4 & 0 \\ 0 & 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

2. *(10 pts)* The rows of the 5x5 nonsingular matrices A and B have the following in common:

   (a) B_1 is A_1 plus 2 times A_2.

   (b) B_2 is 5 times A_5 minus 6 times A_3.

   (c) B_3 is 8 times A_2 minus A_1.

   (d) B_4 is 3 times A_5 minus A_4.

   (e) B_5 is A_4.

   Find the unique matrix C that satisfies $CA = B$.

3. *(15 pts)* Find the inverse matrix for the matrix N below.

$$N = \begin{pmatrix} 5 & 2 & 2 \\ 1 & -5 & 0 \\ -3 & 2 & -1 \end{pmatrix}$$

4. *(15 pts)* Show that the square matrix A is nonsingular ($\text{rref}(A) = I$) if and only if A is invertible.

5. *(15 pts)* Use permutations to compute the determinant of the matrix

$$\begin{pmatrix} 5 & -6 & 10 \\ 1 & 3 & -2 \\ 2 & 4 & 1 \end{pmatrix}$$

6. *(15 pts)* We consider here the matrix R below. Compute the entry in the 3rd row and 4th column of the matrix R^{-1}.

$$R = \begin{pmatrix} 2 & 2 & 2 & 3 \\ 1 & 1 & 1 & 1 \\ 1 & 3 & 2 & 6 \\ 4 & -2 & -4 & 1 \end{pmatrix}$$

7. *(15 pts)* You and your friend Bob are considering the collection $\{\vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4\}$ of vectors in $\mathbb{R}^5$, where

$$\vec{v}_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 0 \\ 4 \end{bmatrix} \quad \vec{v}_2 = \begin{bmatrix} 0 \\ 2 \\ 4 \\ 5 \\ 7 \end{bmatrix} \quad \vec{v}_3 = \begin{bmatrix} 2 \\ 0 \\ 1 \\ 5 \\ 12 \end{bmatrix} \quad \vec{v}_4 = \begin{bmatrix} 3 \\ -4 \\ 0 \\ 1 \\ 10 \end{bmatrix}$$

Bob observes, correctly, that none of these four vectors is a nontrivial linear combination of the other vectors. He cites this fact *alone* as demonstrating that this collection of vectors is linearly independent.

Is Bob's reasoning correct? Explain why or why not. Is Bob's conclusion correct? Explain why or why not.