EXAM 3

Math 107, 2010-2011 Fall, Clark Bray.

You have 50 minutes.

No notes, no books, no calculators.

YOU MUST SHOW ALL WORK AND EXPLAIN ALL REASONING
TO RECEIVE CREDIT. CLARITY WILL BE CONSIDERED IN GRADING.
All answers must be simplified. All of the policies and guidelines
on the class webpages are in effect on this exam.

Good luck!

Name ________________________________

ID number______________________

"I have adhered to the Duke Community Standard in completing this examination."

Signature: ______________________

1. ___________

2. ___________

3. ___________

4. ___________

5. ___________

Total Score ___________ (/100 points)

1. *(15 pts)* Find a particular solution to the system below.

$$\begin{aligned} y_1' &= 3y_1 - 2y_2 + 4x \\ y_2' &= 2y_1 - y_2 + 3 \end{aligned}$$

2. *(20 pts)* We consider here the inner product space $C^0[0, 2]$ using the L^2 inner product. Let V be the subspace spanned by the functions $f_1 = 3$, $f_2 = x^2$. Use the Gram-Schmidt process to find an orthonormal basis for V.

3. *(15 pts)* The matrix A has eigenvalues 5, 2, and 7, and corresponding eigenvectors $(1, 2, 1)$, $(3, -1, -2)$, $(2, 1, 0)$. Find a fundamental set of solutions to the system of equations

$$\vec{y}' = A\vec{y}$$

and also find the solution satisfying the initial condition $\vec{y}(0) = (3, 2, 5)$.

4. *(30 pts)* We consider here the system of equations $\vec{y}' = A\vec{y}$, with

$$A = \begin{pmatrix} 5 & -1 & -4 \\ 2 & 2 & -4 \\ 2 & -1 & 0 \end{pmatrix}$$

The eigenvalues are 3 and 2, and the characteristic polynomial is $p(\lambda) = \lambda^3 - 7\lambda^2 + 16\lambda - 12$. Find a fundamental set of solutions to this system of equations.

(extra space if needed)

5. *(20 pts)* The 3×3 matrix A has eigenvectors

$$\vec{v}_1 = \begin{bmatrix} 2 \\ 3 \\ 6 \end{bmatrix} \quad \vec{v}_1 = \begin{bmatrix} 6 \\ 2 \\ -3 \end{bmatrix} \quad \vec{v}_1 = \begin{bmatrix} 3 \\ -6 \\ 2 \end{bmatrix}$$

Show that A must be symmetric. (Make sure to explain all of your reasoning clearly.)