

EXAM 2

Math 107, 2009-2010 Spring, Clark Bray.

You have 50 minutes.

No notes, no books, no calculators.

YOU MUST SHOW ALL WORK AND EXPLAIN ALL REASONING
TO RECEIVE CREDIT. CLARITY WILL BE CONSIDERED IN GRADING.

Good luck!

Name _____

ID number _____

“I have adhered to the Duke Community
Standard in completing this
examination.”

1. _____

2. _____

3. _____

4. _____

5. _____

6. _____

Signature: _____

Total Score _____ (/100 points)

1. (15 pts) Prove or disprove: $V = \left\{ f \in C^3[a, b] \mid \int_a^b f(x) dx + f'''(m) = 0 \text{ (where } m \in (a, b)) \right\}$ is a subspace of $C^3[a, b]$.

2. (15 pts) Determine if the following functions are independent or dependent:

$$f_1(x) = \sin\left(x + \frac{\pi}{6}\right) \quad f_2(x) = \sin\left(x + \frac{\pi}{4}\right) \quad f_3(x) = \sin\left(x + \frac{\pi}{3}\right)$$

3. (15 pts) A bath has a constant temperature of 40°C , and at time $t = 0$ a metal rod is immersed in the bath. The initial temperature of the rod is 13°C , and after one minute the temperature is 22. Assuming that the temperature follows Newton's law of cooling, find the temperature as a function of time, and determine what time the rod will reach 32°C .

4. (20 pts) Find a fundamental set of solutions to the differential equation

$$y''' - 11y'' + 26y' - 16y = 0$$

(Hint: One of the solutions is $y(x) = e^{8x}$.)

5. (20 pts) A forced and damped oscillation is described by the differential equation

$$u'' + 3u' + 2u = \cos(3t)$$

Compute (directly, without using a memorized formula) the gain in this system (the ratio of the amplitudes of the forcing term and the particular solution).

6. (15 pts) Find a particular solution to the differential equation

$$y'' + 4y = \sin(2x)$$