EXAM 1

Math 107, 2009-2010 Spring, Clark Bray.

You have 50 minutes.

No notes, no books, no calculators.

YOU MUST SHOW ALL WORK AND EXPLAIN ALL REASONING
TO RECEIVE CREDIT. CLARITY WILL BE CONSIDERED IN GRADING.

Good luck!

Name _____

ID number_____

1. _____
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"I have adhered to the Duke Community Standard in completing this examination."

Signature: _____

Total Score _____ (/100 points)

1. *(15 pts)* Consider the matrices A and B defined by

$$A = \begin{pmatrix} 1 & 17 & 6 & 7 \\ 4 & 2 & 8 & 4 \\ 5 & 8 & 10 & 2 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 7 & 1 & 0 & -3 \\ -2 & 5 & 0 & 2 \\ -5 & 7 & 0 & 10 \end{pmatrix}$$

Find a matrix C for which $A = CB$, or show that such a matrix C does not exist.

2. *(15 pts)* Find the inverse of the matrix A defined by

$$A = \begin{pmatrix} 3 & -4 & 7 \\ 0 & 0 & -1 \\ -2 & 3 & 4 \end{pmatrix}$$

3. *(15 pts)* Use permutations to compute the determinant of the matrix M given below. (Do NOT use another method.)

$$M = \begin{pmatrix} 4 & 1 & -1 \\ 3 & 2 & 2 \\ 7 & 5 & 0 \end{pmatrix}$$

4. *(15 pts)* Use pivots to show that it is not possible for any collection of 4 vectors to span $\mathbb{R}^5$. (Make sure to explain all of the steps in your reasoning.)

5. *(15 pts)* Show that in any vector space V, the zero vector $\vec{0}$ is unique.

6. *(10 pts)* We will call a square matrix "spinny" if rotating the matrix by a quarter turn does not change the matrix. For example, the matrix A below is spinny:

$$A = \begin{pmatrix} 1 & 3 & 2 & 1 \\ 2 & 0 & 0 & 3 \\ 3 & 0 & 0 & 2 \\ 1 & 2 & 3 & 1 \end{pmatrix}$$

Show that every 3×3 spinny matrix must have determinant zero. Is this also true for 4×4 spinny matrices? Prove or find a counterexample.

7. *(15 pts)* Write the matrix A below as a product of elementary matrices.

$$A = \begin{pmatrix} 1 & 3 \\ 2 & 2 \end{pmatrix}$$