EXAM 1

Math 212, 2023 Summer Term 2, Clark Bray.

Name:____ Section:______ Student ID:____________

GENERAL RULES

YOU MUST SHOW ALL WORK AND EXPLAIN ALL REASONING TO RECEIVE CREDIT. CLARITY WILL BE CONSIDERED IN GRADING.

No notes, no books, no calculators.

All answers must be reasonably simplified.

All of the policies and guidelines on the class webpages are in effect on this exam.

WRITING RULES

Do not remove the staple, tear pages out of the staple, or tamper with the exam packet in any way. Do not write anything near the staple – this may be cut off.

Use black pen only. You may use a pencil for initial sketches of diagrams, but the final sketch must be drawn over in black pen and you must wipe all erasure residue from the paper.

Work for a given question can be done ONLY on the front or back of the page the question is written on. Room for scratch work is available on the back of this cover page, and on the two blank pages at the end of this packet; scratch work will NOT be graded.

DUKE COMMUNITY STANDARD STATEMENT

"I have adhered to the Duke Community Standard in completing this examination."

Signature: _________________________

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1. *(25 pts)*

   (a) Show that the vectors $\vec{a} = (2, 3, 6)/7$ and $\vec{b} = (3, -6, 2)/7$ are orthogonal unit vectors.

   (b) Use methods from this course to find a third unit vector $\vec{c}$ such that all of $\vec{a}$, $\vec{b}$, $\vec{c}$ are orthogonal to each other.

   (c) Find an equation for the plane P that is parallel to $\vec{a}$ and $\vec{b}$ and contains $\vec{d} = (1, 1, 1)$.

   (d) Find a parametrization for the plane P.

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2. *(20 pts)*

(a) A curve parametrized by $\vec{x}(t)$ has velocity $\vec{v}(t) = (e^t, \sin(3t), 6t^2)$ and starts at $\vec{x}(0) = (1, 2, 3)$. Find an explicit expression for $\vec{x}(t)$.

(b) The surface S has equation $y^2 - x^2 - z^2 + 1 = 0$. It can be viewed as a rotation around an axis A of a curve C in a plane P. Identify A and P, and find an equation for C.

(c) The surface M has equation $2y^2 - (x-3)^2 - (z+4)^2 + 1 = 0$. Describe a geometric process that would turn S into M.

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3. *(20 pts)*

(a) The surface S has equation $xe^x - xy + y^2 z + z = 15$. Find a function f (domain, target, formula) whose graph is S.

(b) The plane $y = 2$ intersects S in a curve C that includes the point $\vec{p} = (0, 2, 3)$. Use the function f from (a) to find the slope of the tangent line to C at $\vec{p}$.

(c) Find a vector perpendicular to S at $\vec{p}$.

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4. *(15 pts)* The physical quantity described by the variable z is easier to compute with in terms of variables s and t, but the Laplacian describing its behavior in the physical world must be in terms of x and y. Specifically we have $z = f(s,t)$ (f is C^2), $s = 2x - 3y$, and $t = x - 5y$.

   Find a fully simplified expression for the Laplacian $\Delta f = \dfrac{\partial^2 z}{\partial x^2} + \dfrac{\partial^2 z}{\partial y^2}$ that is entirely in terms of z and its various partials with respect to s and t.

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5. *(20 pts)* Map location in a given region is described by x and y, and altitude is $z = h(x, y) = x^3 - xy$. At a given moment you are at the point $\vec{p} = (1, 2)$ and moving in the direction parallel to $\vec{v} = (3, 4)$.

(a) What is your rate of change of altitude with respect to distance travelled in that direction?

(b) At $\vec{p}$, what direction points directly uphill?

(c) How steep is the ground at $\vec{p}$?

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