EXAM 1

Math 212, 2020 Fall.

Name:________________________________ NetID:__________ Student ID:__________

GENERAL RULES

YOU MUST SHOW ALL WORK AND EXPLAIN ALL REASONING TO RECEIVE CREDIT. CLARITY WILL BE CONSIDERED IN GRADING.

No calculators.

All answers must be reasonably simplified.

All of the policies and guidelines on the class webpages are in effect on this exam.

It is strongly advised that you use black pen only, so that your work will scan as clearly as possible.

DUKE COMMUNITY STANDARD STATEMENT

"I have adhered to the Duke Community Standard in completing this examination."

Signature: ______________________

(Scratch space. Nothing on this page will be graded!)

1. *(18 pts)* We consider the vectors $\vec{v} = (1, 3, 0)$, $\vec{w} = (0, 2, 1)$, $\vec{u} = (1, 0, 1)$.

   (a) Find the vector $\vec{x} = k\vec{v}$ (where k is constant) for which $\vec{x} - \vec{w}$ is orthogonal to $\vec{w}$.

   (b) Compute the volume of the parallelepiped with edge vectors $\vec{v}$, $\vec{w}$, $\vec{u}$.

   (c) There are six ways to order the vectors $\vec{v}$, $\vec{w}$, $\vec{u}$. Identify which of these six are right-hand orders with a brief explanation. (For full credit you must do this with no redundant arithmetic.)

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2. *(18 pts)* The curve C is entirely within a single plane P, parametrized with position given as $\vec{x}(t)$, and with velocity $\vec{v}(t)$ and acceleration $\vec{a}(t)$.

   (a) Suppose we know only that $\vec{x}(0) = (2, -1, 0)$, $\vec{v}(0) = (-1, 0, 1)$, and $\vec{a}(0) = (-2, -1, 3)$. Find the equation of the plane P. *(Hint: What relationship must $\vec{v}$ and $\vec{a}$ have with P?)*

   (b) Suppose we know further that $\vec{v}(t) = (-2\sin t - \cos t, \cos t - e^t, e^t + 2\sin t)$. Find expressions for $\vec{x}(t)$ and $\vec{a}(t)$.

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3. *(16 pts)*

(a) The curve C is parametrized by $\vec{x}(t) = (1 - t, 2t, 3t)$. Find the equation of the surface S obtained by rotating C around the z-axis. *(Hint: Find an expression for distance from a point on this parametrized curve to the z-axis, as a function of z.)*

(b) The curve P is parametrized by $\vec{x}(t) = (2 + t, 3 - t, e^t)$. The surface R is formed by all of the translations of P in the y-direction (by vectors of the form $(0, k, 0)$). Find a function f whose graph is R.

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4. *(18 pts)* In this question we consider the function $f : \mathbb{R}^3 \to \mathbb{R}^1$ given by $f(x, y, z) = xy + e^{yz}$.

   (a) Based on knowledge from the lectures, explain why f is continuously differentiable, noting all important details in the argument.

   (b) Find a function h of the form $h(x, y, z) = c_1 + c_2x + c_3y + c_4z$ that has the same value and partial derivatives as f at the point $(3, 1, 0)$.

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5. *(18 pts)*

(a) The function $p : \mathbb{R}^3 \to \mathbb{R}^3$ is given by $p(x, y, z) = (xe^y, y - z\cos x, xz - \sin z)$. A particle in the domain is moving as described by $\vec{x}(t) = (x(t), y(t), z(t)) = (1-t, 2+t, 3+4t)$. Find the velocity of the image $p(\vec{x}(t))$ at $t = 0$ WITHOUT finding an explicit expression for $p(\vec{x}(t))$.

(b) The variable w is a twice continuously differentiable function of x, y, and z. Also, x, y, and z (unrelated to part (a) above) are functions of s and t given as $x = s - t$, $y = 2s + t$, and $z = 3t - s$. Find a fully simplified expression for

$$\frac{\partial}{\partial t}\left(s\, y\, w_x\right)$$

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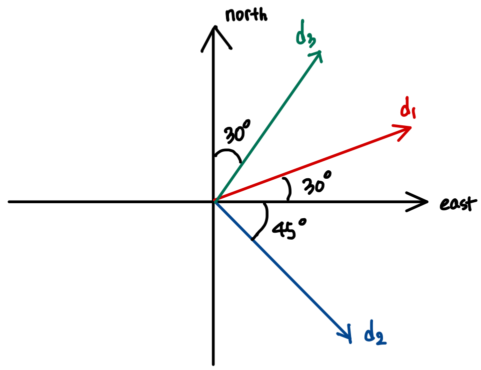

6. *(12 pts)* You are standing at a certain point $\vec{a}$ on a smooth hill. As you face in the direction d_1 (see the figure above) the slope in that direction is 0.3 uphill; as you face in the direction d_2 the slope in that direction is 0.2 downhill.

   Suppose that from this point you start walking in the direction d_3. How steep will your path on the hill be initially?

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