## EXAM 3

Math 212, 2020 Spring, Clark Bray.

Name: $\qquad$ Section: $\qquad$ Student ID: $\qquad$

## GENERAL RULES

YOU MUST SHOW ALL WORK AND EXPLAIN ALL REASONING TO RECEIVE CREDIT. CLARITY WILL BE CONSIDERED IN GRADING.

No calculators.
All answers must be reasonably simplified.
All of the policies and guidelines on the class webpages are in effect on this exam.

## WRITING RULES

Use black pen only. You may use a pencil for initial sketches of diagrams, but the final sketch must be drawn over in black pen and you must wipe all erasure residue from the paper.

## DUKE COMMUNITY STANDARD STATEMENT

"I have adhered to the Duke Community Standard in completing this examination."

Signature: $\qquad$
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1. (20 pts) As a function of location $(x, y)$, the wind applies a force given by

$$
\vec{F}(x, y)=(3-x+2 y, 3 x+y-2)
$$

The curve $C$ is the circle of radius 4 centered at the origin. Compute the amount of work that is required to move clockwise along $C$ from $(0,4)$ to $(0,-4)$.
(extra space for questions from other side)
2. (20 pts) The curve $P$ starts at $(1,0,0)$, then moves in quarter circle arcs to $(0,0,1)$, then $(0,1,0)$, then back to $(1,0,0)$, as parametrized by

$$
\vec{x}(t)=\left\{\begin{aligned}
(\cos t, 0, \sin t) & 0 & \leq t \leq \pi / 2 \\
(0,-\cos t, \sin t) & \pi / 2 & \leq t \leq \pi \\
(-\sin t,-\cos t, 0) & \pi & \leq t \leq 3 \pi / 2
\end{aligned}\right.
$$

Compute the line integral along $P$ of the vector field

$$
\vec{F}(x, y, z)=\left(\begin{array}{c}
x^{2} e^{x} \\
2-\sin \pi y \\
x^{2}+y^{2}-z^{2}
\end{array}\right)
$$

(extra space for questions from other side)
3. (20 pts) The surface $S$ is the octahedron with vertices at $( \pm 1,0,0),(0, \pm 1,0),(0,0, \pm 1)$, oriented toward the origin. Compute the flux through $S$ of the vector field

$$
\vec{G}(x, y, z)=\left(\begin{array}{c}
3 x y+e^{y}-1 \\
x e^{x}-3 z^{3} \\
x^{2} y-3 z
\end{array}\right)
$$

(extra space for questions from other side)
4. (20 pts) The surface $H$ is the part of $x^{2}+z^{2}=e^{y}$ with $0 \leq y \leq 2$, oriented away from the $y$-axis. Compute the flux through $H$ of the vector field

$$
\vec{M}(x, y, z)=\left(\begin{array}{c}
z-2 x y \\
y^{2}+2 \\
x^{2}+y^{2}
\end{array}\right)
$$

(extra space for questions from other side)
5. (20 pts) Find the "highest point" (greatest value of $z$ ) on the rotated ellipsoid with equation $3 x^{2}-2 x y+y^{2}+4 z^{2}+x z=3$.
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