

EXAM 2

Math 212, 2020 Spring, Clark Bray.

Name: Solutions Section: _____ Student ID: _____

GENERAL RULES

YOU MUST SHOW ALL WORK AND EXPLAIN ALL REASONING TO RECEIVE CREDIT.
CLARITY WILL BE CONSIDERED IN GRADING.

No notes, no books, no calculators.

All answers must be reasonably simplified.

All of the policies and guidelines on the class webpages are in effect on this exam.

WRITING RULES

Do not write anything near the staple – this will be cut off.

Use black pen only. You may use a pencil for initial sketches of diagrams, but the final sketch must be drawn over in black pen and you must wipe all erasure residue from the paper.

Work for a given question can be done ONLY on the front or back of the page the question is written on. Room for scratch work is available on the back of this cover page, and on the two blank pages at the end of this packet; scratch work will NOT be graded.

DUKE COMMUNITY STANDARD STATEMENT

“I have adhered to the Duke Community Standard in completing this examination.”

Signature: _____

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1. (20 pts) The surface S has equation $z^3 - xz^2 - 2xyz = 3xy$. At the point $(1, 2, 3)$ on S , compute the slope of the tangent line to the cross section of S in the plane $x = 1$.

$$F(x, y, z) = 0, \text{ with } F(x, y, z) = z^3 - xz^2 - 2xyz - 3xy$$

$$\frac{\partial F}{\partial z} = 3z^2 - 2xz - 2xy, \text{ so } \frac{\partial F}{\partial z}(1, 2, 3) = 17 \neq 0$$

So we can view z as a function of x, y .

The slope in question then is $\frac{\partial z}{\partial y}(1, 2)$.

$$\frac{\partial z}{\partial y} = - \frac{\partial F / \partial y}{\partial F / \partial z}$$

$$= - \frac{-2xz - 3x}{3z^2 - 2xz - 2xy}$$

$$= \frac{2xz + 3x}{3z^2 - 2xz - 2xy}$$

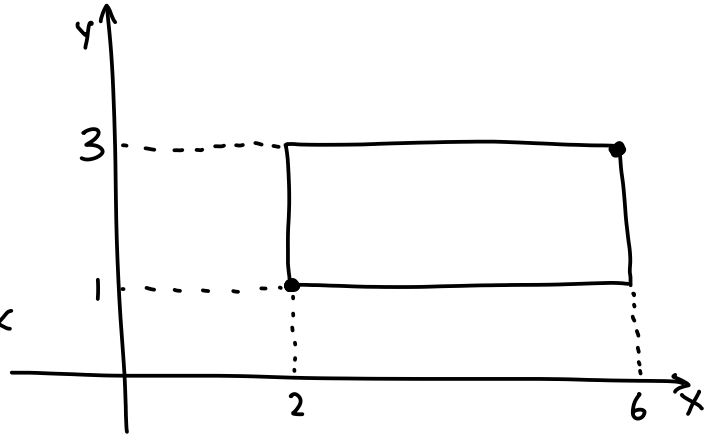
$$\frac{\partial z}{\partial y}(1, 2) = \frac{9}{17}$$

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2. (20 pts) The plot of land P is the rectangle in the xy -plane with opposing vertices at the points $(2, 1)$ and $(6, 3)$. Grass is spread over this plot with density (in millions of blades of grass per unit area) given by $\delta(x, y) = 4xy + e^y$. Compute the total number of blades of grass on P .

$$\text{Pop.} = \iint_P \delta \, dA$$

$$= \int_2^6 \int_1^3 (4xy + e^y) \, dy \, dx$$



$$= \int_2^6 \left(2xy^2 + e^y \right) \Big|_{y=1}^{y=3} dx$$

$$= \int_2^6 (16x + (e^3 - e)) \, dx$$

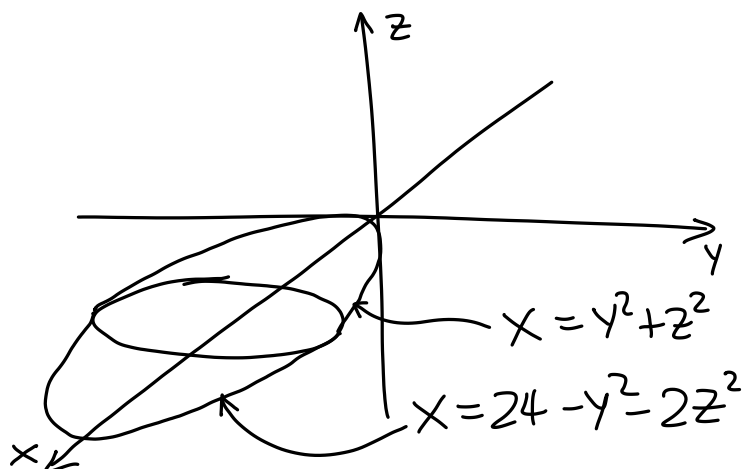
$$= \left(8x^2 + (e^3 - e)x \right) \Big|_{x=2}^{x=6}$$

$$= 256 + 4(e^3 - e)$$

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3. (20 pts) The domain T is bounded by the surfaces $x = y^2 + z^2$ and $x + y^2 + 2z^2 = 24$. Set up (but do not evaluate!) an iterated integral representing $\iiint_T 3x^2 + 3z^2 dV$.

Also, this integral represents the moment of inertia of a mass in T of what density and around what axis?

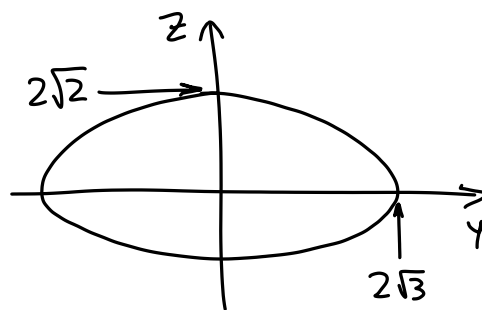


Intersection :

$$y^2 + z^2 = 24 - y^2 - 2z^2$$

$$2y^2 + 3z^2 = 24$$

$$\left(\frac{y}{2\sqrt{3}}\right)^2 + \left(\frac{z}{2\sqrt{2}}\right)^2 = 1$$



$$\iiint_T 3(x^2 + z^2) dV$$

$$= \int_{-2\sqrt{3}}^{2\sqrt{3}} \int_{-\sqrt{\frac{24-2y^2}{3}}}^{\sqrt{\frac{24-2y^2}{3}}} \int_{y^2+z^2}^{24-y^2-2z^2} 3(x^2 + z^2) dx dz dy$$

(or, equiv.)

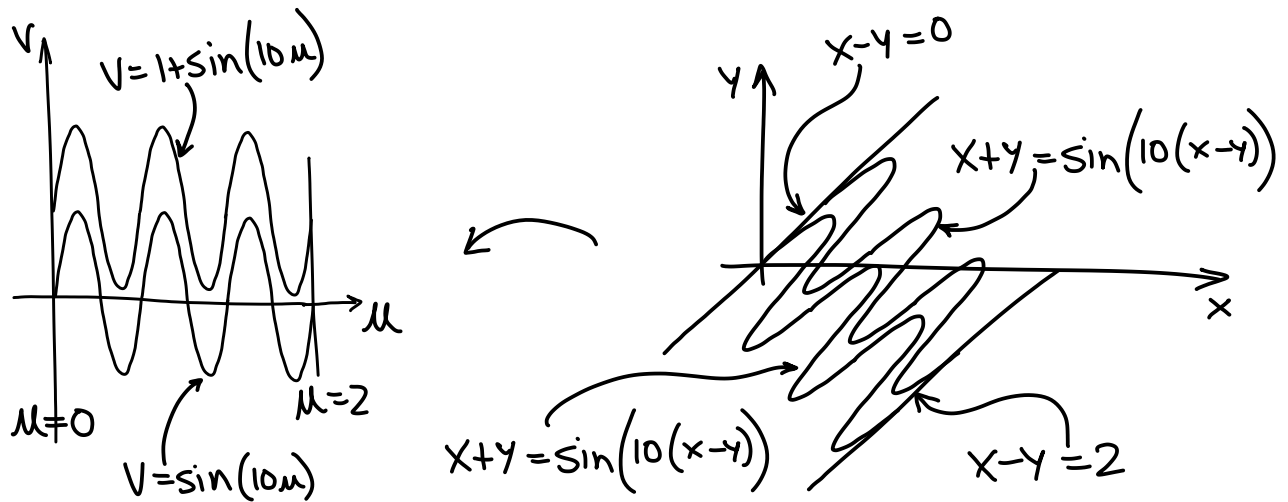
$$= \int_{-2\sqrt{2}}^{2\sqrt{2}} \int_{-\sqrt{\frac{24-3z^2}{2}}}^{\sqrt{\frac{24-3z^2}{2}}} \int_{y^2+z^2}^{24-y^2-2z^2} 3(x^2 + z^2) dx dy dz$$

$I = 3$, around y -axis.

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4. (25 pts) The domain D in the xy -plane is bounded by the curves $x + y = \sin(10(x - y))$, $x + y = 1 + \sin(10(x - y))$, $x - y = 0$, and $x - y = 2$. Compute $\iint_D 6x - 6y \, dx \, dy$.

Substitute $u = x - y$, $v = x + y$:



$$\frac{\partial(u,v)}{\partial(x,y)} = \det \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} = 2, \quad \left| \frac{\partial(x,y)}{\partial(u,v)} \right| = \left| \frac{1}{2} \right| = \frac{1}{2}$$

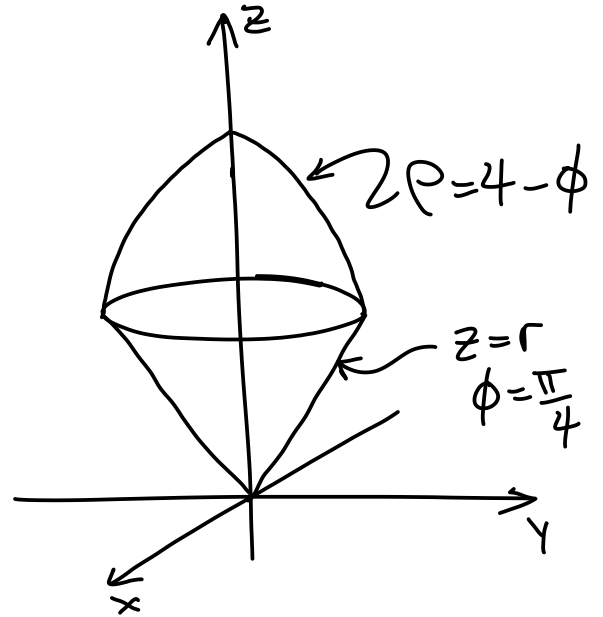
$$\begin{aligned} \iint_D 6x - 6y \, dx \, dy &= \iint (6x - 6y) \left| \frac{\partial(x,y)}{\partial(u,v)} \right| du \, dv \\ &= \iint 3(x - y) \, du \, dv = \iint 3u \, du \, dv \\ &= 3 \int_0^2 \int_{\sin(10u)}^{1 + \sin(10u)} u \, dv \, du = 3 \int_0^2 (uv) \Big|_{v=\sin(10u)}^{v=1+\sin(10u)} du \\ &= 3 \int_0^2 u \, du = 3 \left(\frac{1}{2} u^2 \right) \Big|_0^2 = 6 \end{aligned}$$

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5. (15 pts) The surface S has spherical equation $\rho = 4 - \phi$, for $0 \leq \phi \leq \pi$. The solid R is the region inside of S and above the cone with cylindrical equation $z = r$. Set up (but do not evaluate!) a triple iterated integral for the volume of R .

$$V = \iiint_R 1 \, dV$$

$$= \iiint (1) \rho^2 \sin \phi \, d\rho \, d\phi \, d\theta$$



$$= \int_0^{2\pi} \int_0^{\pi/4} \int_0^{4-\phi} \rho^2 \sin \phi \, d\rho \, d\phi \, d\theta$$

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