EXAM 3

Math 212, 2019 Fall, Clark Bray.

Name:_____ Section:_____ Student ID:_____

GENERAL RULES

YOU MUST SHOW ALL WORK AND EXPLAIN ALL REASONING TO RECEIVE CREDIT. CLARITY WILL BE CONSIDERED IN GRADING.

No notes, no books, no calculators.

All answers must be reasonably simplified.

All of the policies and guidelines on the class webpages are in effect on this exam.

WRITING RULES

Do not write anything near the staple – this will be cut off.

Use black pen only. You may use a pencil for initial sketches of diagrams, but the final sketch must be drawn over in black pen and you must wipe all erasure residue from the paper.

Work for a given question can be done ONLY on the front or back of the page the question is written on. Room for scratch work is available on the back of this cover page, and on the two blank pages at the end of this packet; scratch work will NOT be graded.

DUKE COMMUNITY STANDARD STATEMENT

"I have adhered to the Duke Community Standard in completing this examination."

Signature: _____

1. *(20 pts)* The curve C is the intersection of the unit sphere with the plane $x + z = 0$, oriented counterclockwise as seen from above. The curve D is the part of C that is on or above the xy-plane. The vector field $\vec{F}$ is given below.

$$\vec{F}(x, y, z) = \begin{pmatrix} 2xy \\ x^2 + 3y^2 - z^2 \\ -2yz \end{pmatrix}$$

Compute $\int_D \vec{F} \cdot d\vec{x}$.

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2. *(20 pts)* The surface P is the part of the plane $2x - 2y + 4z = 9$ (oriented in the positive direction along the y-axis) whose projection to the yz-plane is the rectangle with $y \in [0, 1]$ and $z \in [1, 2]$. The vector field $\vec{G}$ is given below.

$$\vec{G}(x, y, z) = \begin{pmatrix} 3 \\ y + z \\ x \end{pmatrix}$$

Compute $\iint_P \vec{G} \cdot d\vec{S}$.

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3. *(20 pts)* The solid region M is bounded by the tetrahedron with vertices at $(1, 0, 1)$, $(1, 0, -1)$, $(-1, 1, 0)$, $(-1, -1, 0)$. The vector field $\vec{H}$, representing a flow of a fluid in the vicinity, is given below.

$$\vec{H}(x, y, z) = \begin{pmatrix} xy^3 \\ xz \\ 4 - 3y \end{pmatrix}$$

Compute the rate of change of the quantity of this fluid in M.

4. *(20 pts)* The curve C is a square with vertices $(0,0,0)$, $(2,-3,6)$, $(3,6,2)$, $(5,3,8)$, oriented clockwise as seen from above. The vector field $\vec{F}$, representing the flow of a fluid, is given below.

$$\vec{F}(x,y,z) = \begin{pmatrix} x - 2y + 3z \\ 3x + 5y - z \\ 6x + 2y + 4z \end{pmatrix}$$

Compute the circulation of this fluid around C. *(Hint: You might find a use for the vector (-6,2,3).)*

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5. *(20 pts)* The inward-oriented tetrahedron T has vertices $(1, 0, 0)$, $(0, 1, 0)$, $(0, 0, 1)$, $(0, 0, 0)$, and S is made up of the three of those faces touching the vertex $(0, 1, 0)$. The vector field $\vec{G}$ is given below.

$$\vec{G}(x, y, z) = \begin{pmatrix} 3 - xz \\ 6 + yz \\ 4 - xy \end{pmatrix}$$

Compute $\iint_S \vec{G} \cdot d\vec{S}$.

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