## EXAM 2

Math 212, 2019 Spring, Clark Bray.

Name:	Section:	Student ID:
GENERAL I	RULES	
YOU MUST SHOW ALL WORK AND EXPLAIN A CLARITY WILL BE CONSIDERED IN GRADING.		G TO RECEIVE CREDIT.
No notes, no books, no calculators.		
All answers must be reasonably simplified.		
All of the policies and guidelines on the class webpage	es are in effect on	this exam.
WRITING 1	RULES	
Do not write anything on the QR codes or nearby pri	int, or near the st	aple.
Use black pen only. You may use a pencil for initial sk drawn over in black pen and you must wipe all erasur	~	
Work for a given question can be done ONLY on the on. Room for scratch work is available on the back of the end of this packet; scratch work will NOT be grade	f this cover page,	
DUKE COMMUNITY STA	NDARD STAT	EMENT
"I have adhered to the Duke Community Sta	ındard in complet	ing this examination."
Signature:		

1. (20 pts) The function  $f: \mathbb{R}^3 \to \mathbb{R}^3$  is given by

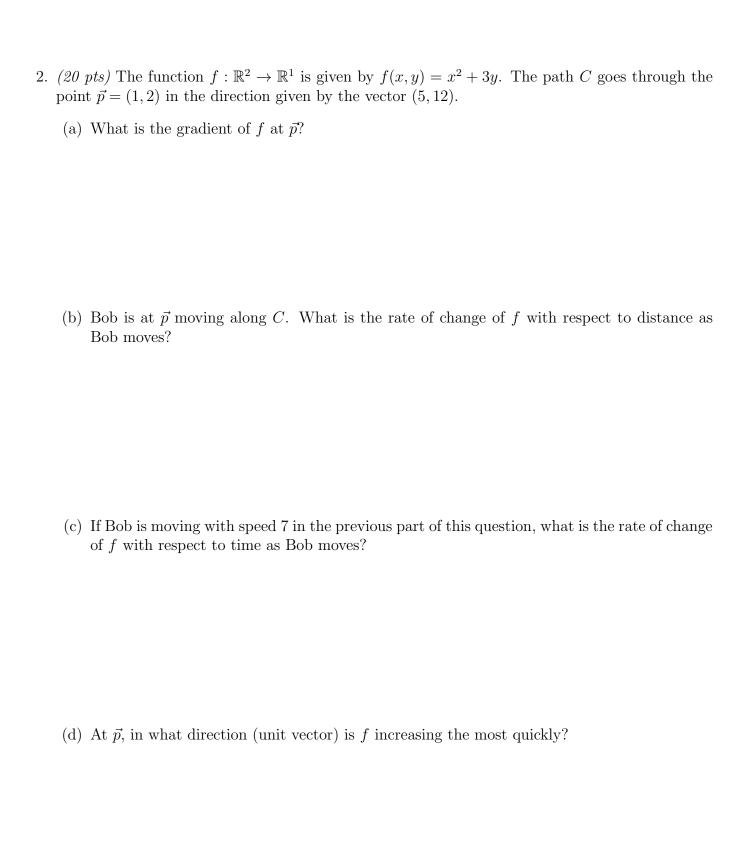
$$f\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3xy - z^2 \\ e^x z \\ \cos e^y \end{pmatrix} = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$$

(a) Find an expression for the Jacobian matrix of f.

(b) The function  $g: \mathbb{R}^3 \to \mathbb{R}^3$  is given by

$$g\begin{pmatrix} r\\s\\t \end{pmatrix} = \begin{pmatrix} r-s\\1+s-t\\r+s+2t \end{pmatrix}$$

Compute the Jacobian matrix for the composition  $h = f \circ g$  at the point (r, s, t) = (0, 0, 1) WITHOUT identifying the function h itself.



3. (20 pts) The solid tetrahedron T has vertices at (0,0,0), (2,0,0), (1,2,0), and (1,2,3). Write a triple iterated integral (just one!) that represents the mass in T, with density given as  $\delta(x,y,z) = xe^y$ . (You can use any coordinate system you choose, and do not have to evaluate the iterated integral.)

4. (20 pts) Recall that the function R below rotates points (u, v) in the plane counterclockwise around the origin by the angle  $\theta$ .

$$R(u,v) = \begin{pmatrix} u\cos\theta - v\sin\theta\\ u\sin\theta + v\cos\theta \end{pmatrix}$$

Use this to compute the integral  $\iint_D x \, dx \, dy$ , where D is the rectangle with vertices at (0,0), (4,3), (-3,4), and (1,7).

5. (20 pts) The solid M is the part of the first octant with  $y \ge x$  and  $x^2 + (y-1)^2 + z^2 \le 1$ . Write a triple iterated integral (just one!) that represents the mass in M, with density given as  $\delta(x,y,z) = z$ . (You can use any coordinate system you choose, and do not have to evaluate the iterated integral.)