# EXAM 2

Math 212, 2019 Spring, Clark Bray.

Name:________________________________________ Section:______ Student ID:____________

## GENERAL RULES

YOU MUST SHOW ALL WORK AND EXPLAIN ALL REASONING TO RECEIVE CREDIT. CLARITY WILL BE CONSIDERED IN GRADING.

No notes, no books, no calculators.

All answers must be reasonably simplified.

All of the policies and guidelines on the class webpages are in effect on this exam.

## WRITING RULES

Do not write anything on the QR codes or nearby print, or near the staple.

Use black pen only. You may use a pencil for initial sketches of diagrams, but the final sketch must be drawn over in black pen and you must wipe all erasure residue from the paper.

Work for a given question can be done ONLY on the front or back of the page the question is written on. Room for scratch work is available on the back of this cover page, and on the two blank pages at the end of this packet; scratch work will NOT be graded.

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## DUKE COMMUNITY STANDARD STATEMENT

"I have adhered to the Duke Community Standard in completing this examination."

Signature: ________________________

*(Scratch space. Nothing on this page will be graded!)*

1. *(20 pts)* The function $f : \mathbb{R}^3 \to \mathbb{R}^3$ is given by

$$f\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3xy - z^2 \\ e^x z \\ \cos e^y \end{pmatrix} = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$$

(a) Find an expression for the Jacobian matrix of $f$.

(b) The function $g : \mathbb{R}^3 \to \mathbb{R}^3$ is given by

$$g\begin{pmatrix} r \\ s \\ t \end{pmatrix} = \begin{pmatrix} r - s \\ 1 + s - t \\ r + s + 2t \end{pmatrix}$$

Compute the Jacobian matrix for the composition $h = f \circ g$ at the point $(r, s, t) = (0, 0, 1)$ WITHOUT identifying the function $h$ itself.

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2. *(20 pts)* The function $f : \mathbb{R}^2 \to \mathbb{R}^1$ is given by $f(x, y) = x^2 + 3y$. The path $C$ goes through the point $\vec{p} = (1, 2)$ in the direction given by the vector $(5, 12)$.

   (a) What is the gradient of $f$ at $\vec{p}$?

   (b) Bob is at $\vec{p}$ moving along $C$. What is the rate of change of $f$ with respect to distance as Bob moves?

   (c) If Bob is moving with speed 7 in the previous part of this question, what is the rate of change of $f$ with respect to time as Bob moves?

   (d) At $\vec{p}$, in what direction (unit vector) is $f$ increasing the most quickly?

*(extra space for question from other side)*

3. *(20 pts)* The solid tetrahedron $T$ has vertices at $(0,0,0)$, $(2,0,0)$, $(1,2,0)$, and $(1,2,3)$. Write a triple iterated integral (just one!) that represents the mass in $T$, with density given as $\delta(x,y,z) = xe^y$. (You can use any coordinate system you choose, and do not have to evaluate the iterated integral.)

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4. *(20 pts)* Recall that the function $R$ below rotates points $(u, v)$ in the plane counterclockwise around the origin by the angle $\theta$.

$$R(u, v) = \begin{pmatrix} u\cos\theta - v\sin\theta \\ u\sin\theta + v\cos\theta \end{pmatrix}$$

Use this to compute the integral $\iint_D x\,dx\,dy$, where $D$ is the rectangle with vertices at $(0, 0)$, $(4, 3)$, $(-3, 4)$, and $(1, 7)$.

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5. *(20 pts)* The solid $M$ is the part of the first octant with $y \geq x$ and $x^2 + (y-1)^2 + z^2 \leq 1$. Write a triple iterated integral (just one!) that represents the mass in $M$, with density given as $\delta(x, y, z) = z$. (You can use any coordinate system you choose, and do not have to evaluate the iterated integral.)

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