## EXAM 3

Math 212, 2017-2018 Spring, Clark Bray.

YOU MUST SHOW ALL WORK AND EXPLAIN ALL REASONING TO RECEIVE CREDIT.
CLARITY WILL BE CONSIDERED IN GRADING.

No notes, no books, no calculators.
All answers must be simplified.

All of the policies and guidelines on the class webpages are in effect on this exam.

Do not write anything on the QR codes or near the staple.

Work for a given question can ONLY be done on the front or back of the page the question is written on.
"I have adhered to the Duke Community Standard in completing this examination."

Signature: $\qquad$
(Nothing on this page will be graded!)

1. (20 pts) Position on the side of a particular hill is given by distance (in meters) east ( $x$ ) and north (y) of a fixed reference point. A combination of wind and steepness exerts a force (in Newtons) on you at a point $(x, y)$ given by $\vec{F}(x, y)=\left(4+y+3 x^{2}, 1+x+6 y^{2}\right)$.
The path you walk, from $t=0$ to $t=1$, is parametrized by $\vec{x}(t)=(10 \cos (\pi t), 5 \sin (3 \pi t / 2))$. Compute the amount of work (in Joules, aka Newton-meters) that you perform in this walk.
(extra space for question from other side)
2. (20 pts) The circle $C_{1}$ is centered at the origin with radius 1 , and the circle $C_{2}$ is centered at the origin with radius 2 .
Compute the line integral $\int_{P} \vec{G} \cdot d \vec{x}$ of the vector field $\vec{G}(x, y)=\left(3 x^{2}+2 y, 5 x-6 y^{2}\right)$ over the path $P$ that starts at $(1,0)$, moves counter clockwise around $C_{1}$ to $(-1,0)$, then in a straight line to $(-2,0)$, then clockwise around $C_{2}$ to $(2,0)$, and then in a straight line to $(1,0)$.
(extra space for question from other side)
3. (20 pts) Compute the flux of the vector field $\vec{F}(\vec{x})=\left(3 x+4 y, 5 x-z, x^{3}+y z\right)$ through the surface $S$ that is the inward oriented boundary of the solid $R$ defined by $\rho \leq 2$ and $z \geq 0$.
(extra space for question from other side)
4. (20 pts) The curve $C$ starts at $(0,0,1)$, and then in straight line segments goes to $(0,1,1),(0,1,0)$, $(1,1,0),(1,0,0),(1,0,1)$, and then back to $(0,0,1)$. Use Stokes's curl theorem to compute the circulation along $C$ of the vector field $\vec{F}(\vec{x})=\left(x^{3}+x y z, y^{3}+x y z, z^{3}+x y z\right)$.
(extra space for question from other side)
5. (20 pts) The surface $S$ is the part of the sphere $\rho=3$ with $x \geq 0$, oriented toward the origin. Compute the flux through $S$ of the vector field $\vec{H}(\vec{x})=\left(2+x^{4}-5 y, x^{2}+3 z,-4 x^{3} z\right)$.
(extra space for question from other side)
