## EXAM 3

Math 212, 2017-2018 Fall, Clark Bray.

You have 50 minutes.
No notes, no books, no calculators.
YOU MUST SHOW ALL WORK AND EXPLAIN ALL REASONING TO RECEIVE CREDIT. CLARITY WILL BE CONSIDERED IN GRADING.

All answers must be simplified. All of the policies and guidelines on the class webpages are in effect on this exam.

Good luck!

Name $\qquad$

> "I have adhered to the Duke Community
> Standard in completing this
> examination."

1. $\qquad$
Signature: $\qquad$
2. $\qquad$
3. $\qquad$
4. $\qquad$
5. $\qquad$

Total Score $\qquad$ (/100 points)

1. (20 pts) The line $L$ is the intersection of the planes $x=2 y$ and $x=3 z$, and the downward oriented curve $C$ is the part of this line that is in the ball centered at the origin and of radius 7 . Find the line integral over this curve $C$ of the vector field $\vec{F}$ below.

$$
\vec{F}(x, y, z)=(y z, x z+3 z, x y+3 y)
$$

2. (20 pts) Compute the line integral of the vector field $\vec{G}$ (below) around the polygonal curve $P$ that starts at the origin, then goes in sequence to $(0,1,1),(5,1,6),(5,-1,4),(0,-1,-1)$, and then back to the origin.

$$
\vec{G}(x, y, z)=\left(\begin{array}{c}
x+2 z \\
3 x-y \\
2 y-5 z
\end{array}\right)
$$

(Hint: All of the edges of $P$ are orthogonal to the vector $(1,1,-1)$.)
3. (20 pts) The curve $G$ is parametrized by $\vec{x}(t)=(\sin t, \cos t, t)$ with $t \in[0, \pi]$. Find the line integral over $G$ of the vector field $\vec{H}$ below.

$$
\vec{H}(x, y, z)=\left(x, x, x^{2} y\right)
$$

4. (20 pts) The region $R$ consists of the points that are both above the cone $z=\sqrt{x^{2}+y^{2}}$ and below the plane $z=4$. At a given moment, the vector field describing the flow of gnats in the vicinity of this region is

$$
\vec{I}(x, y, z)=(3 x-y z, x z+2 y, 8 x z)
$$

To address the question of the momentary rate of change of the total number of gnats in $R$, find the flux of $\vec{I}$ through the inward oriented boundary of $R$.
5. (20 pts) The downward oriented surface $P$ is the part of the paraboloid $z=x^{2}+y^{2}$ that is inside the cylinder $(x-1)^{2}+(y-1)^{2}=2$. Compute the flux through $P$ of the vector field $\vec{W}$ below.

$$
\vec{W}(x, y, z)=\left(\begin{array}{c}
3-e^{z-2 x-2 y} \\
4+e^{z-2 x-2 y} \\
14
\end{array}\right)
$$

