EXAM 1

Math 212, 2017-2018 Fall, Clark Bray.

You have 50 minutes.

No notes, no books, no calculators.

YOU MUST SHOW ALL WORK AND EXPLAIN ALL REASONING
TO RECEIVE CREDIT. CLARITY WILL BE CONSIDERED IN GRADING.
All answers must be simplified. All of the policies and guidelines
on the class webpages are in effect on this exam.

Good luck!

Name _____

"I have adhered to the Duke Community
Standard in completing this
examination."

1. _____

2. _____ Signature: _____

3. _____

4. _____

5. _____

6. _____

Total Score _____ (/100 points)

1. *(18 pts)* In this question we consider the vectors $\vec{v} = (3, 1, 0)$ and $\vec{w} = (1, 0, 2)$.

   (a) Find the angle between these two vectors (leave your answer in terms of an inverse trig function).

   (b) Find the component of $\vec{w}$ in the direction of $\vec{v}$.

   (c) Find the area of the parallelogram P defined by these two vectors.

2. *(15 pts)* Find the equation of the plane S that is parallel to $\vec{v}$ and perpendicular to the parallelogram P (both from question 1), and goes through the point $(0, 0, 1)$.

3. *(18 pts)* A particle with position $\vec{x}(t)$ has velocity $\vec{v}(t) = (2t + 1, e^t, \sin t)$ and $\vec{x}(0) = (1, 2, 3)$.

   (a) Find an expression for $\vec{x}(t)$.

   (b) Let L be the line tangent to the particle's path at time $t = 0$. The symmetric equations for L do not take the usual form due to a zero in the velocity vector; but the usual process of converting the parametrization does still yield the equations of a pair of planes defining L. Find those two equations.

4. *(18 pts)* The surface S has equation $x^2z - 2y^4 + 3z = 0$.

   (a) Is S the graph of a function $f : \mathbb{R}^a \to \mathbb{R}^b$? If not, why? And if so, find a, b, and an expression for f.

   (b) Is S a level set of a function $g : \mathbb{R}^c \to \mathbb{R}^d$? If not, why? And if so, find c, d, and an expression for g.

5. *(16 pts)* Find the equation of the tangent plane to the graph of the function $f(x, y) = 3x^2 + 5y^2$ at the point $(2, 1, 17)$.

6. *(15 pts)* Due to the angle of the sun at a given moment, the shadow in the plane $x + 2y + 5z = 0$ of a particle at position (x, y, z) in space is given by

$$\vec{s}(x, y, z) = \begin{pmatrix} 4x + 6y + 15z \\ 3x + 7y + 15z \\ -2x - 4y - 9z \end{pmatrix}$$

At this moment, the particle is at $(3, 2, 1)$ and moving with velocity $(2, 1, 1)$. What is the velocity of this point's shadow?